EE8010-POWER SYSTEMS TRANSIENTS

OBJECTIVES:

- To impart knowledge about the following topics:
- Generation of switching transients and their control using circuit – theoretical concept.
- Mechanism of lighting strokes and the production of lighting surges.
- Propagation, reflection and refraction of travelling waves.
- Voltage transients caused by faults, circuit breaker action, load rejection on integrated power system.

UNIT-I INTRODUCTION AND SURVEY 9

Review and importance of the study of transients - causes for transients. RL circuit transient with sine wave excitation - double frequency transients - basic transforms of the RLC circuit transients. Different types of power system transients - effect of transients on power systems – role of the study of transients in system planning.

UNIT II SWITCHING TRANSIENTS 9

Over voltages due to switching transients - resistance switching and the equivalent circuit for interrupting the resistor current - load switching and equivalent circuit - waveforms for transient voltage across the load and the switch - normal and abnormal switching transients. Current suppression - current chopping - effective equivalent circuit. Capacitance switching - effect of source regulation - capacitance switching with a restrike, with multiple restrikes. Illustration for multiple restriking transients - ferro resonance.

UNIT III LIGHTNING TRANSIENTS 9

Review of the theories in the formation of clouds and charge formation - rate of charging of thunder clouds – mechanism of lightning discharges and characteristics of lightning strokes – model for lightning stroke - factors contributing to good line design - protection using ground wires - tower footing resistance - Interaction between lightning and power system.

UNIT IV TRAVELING WAVES ON TRANSMISSION LINE COMPUTATION OF TRANSIENTS 9

Computation of transients - transient response of systems with series and shunt lumped parameters and distributed lines. Traveling wave concept - step response - Bewely's lattice diagram - standing waves and natural frequencies - reflection and refraction of travelling waves.

UNIT V TRANSIENTS IN INTEGRATED POWER SYSTEM 9

The short line and kilometric fault - distribution of voltages in a power system – Line dropping and load rejection - voltage transients on closing and reclosing lines – over voltage induced by faults - switching surges on integrated system Qualitative application of EMTP for transient computation.

TOTAL : 45 PERIODS

TEXT BOOKS:

1. Allan Greenwood, 'Electrical Transients in Power Systems', Wiley Inter Science, New York, 2 nd Edition, 1991.
2. Pritindra Chowdhari, "Electromagnetic transients in Power System", John Wiley and Sons Inc.,Second Edition, 2009.
3. C.S. Indulkar, D.P.Kothari, K. Ramalingam, 'Power System Transients – A statistical approach', PHI Learning Private Limited, Second Edition, 2010.

REFERENCES

1. M.S.Naidu and V.Kamaraju, 'High Voltage Engineering', McGraw Hill, Fifth Edition, 2013.
2. R.D. Begamudre, 'Extra High Voltage AC Transmission Engineering', Wiley Eastern Limited, 1986.
3. Y.Hase, Handbook of Power System Engineering," Wiley India, 2012.
4. J.L.Kirtley, "Electric Power Principles, Sources, Conversion, Distribution and use," Wiley, 2012.
5. Akihiro ametani," Power System Transient theory and applications", CRC press, 2013.

UNIT-I

INTRODUCTION AND SURVEY

1.1 Transient:-

* Sinusoidal wave deviates from its normal form whenever there is a sudden change in the system such as by faults, addition/or removal of heavy loads power outages. etc.
* It denotes abrupt change in voltage and current for short duration.
* Voltage transients lasts for 50 μs and current transient last for 20 μs

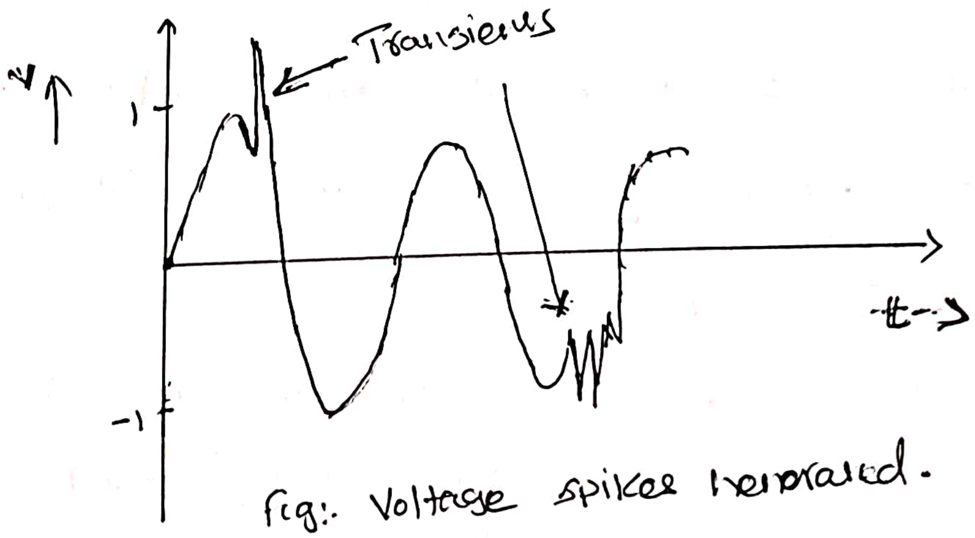


fig:- Voltage spikes generated.

1.2 Review and Importance of study of Transient:-

* Transients are momentary changes taken place in voltage or current waveform that occur over a short interval of time.
* These disturbances occur in a few cycles which are difficult to be identified.

* Transient cause serious disturbances in the reliability safety and economy of power system network.

* So, its very important ~~to~~ first to detect and classify the type of transient and then to mitigate it.

1.2.1 Reason:- for Transient:-

* We know that Any electric circuit composed of ~~the~~ three kind of parameter ie Resistance (R), inductance (L) & Capacitance (C).

* whenever there is sudden change in system condition due to faults, (symmetrical & Unsymmetrical), line energizing, (or) de energizing etc., the current through the inductor cannot be able ~~to~~ changed instantaneously

* ~~In~~ ~~Sam~~ due to energy stored in the inductor

ie $\boxed{E_L = \frac{1}{2} L I^2}$

* Similarly voltage across the capacitor can't be changed instantly due to the energy stored in the capacitor ie $\boxed{E_C = \frac{1}{2} C V^2}$

* So, the circuit takes finite time to change from present state & to new state.

* This finite time is called transient period

* During this time period, the transients remain in the system, and causes considerable damage to the system components. (Transformer, motor failures)

* So, transient to be identified in advance and have to be cleared before it making any damage to the system components.

1.2.2 Effects of Electrical faults and transients:-

Transient results the following in

1. Black-out in a city
2. Shutdown of plant
3. Fires in some buildings
4. Unusual power cuts. etc

1.3 CAUSES OF TRANSIENTS:-

The causes of power system transients can be divided into two categories

(a) External causes [or] Natural causes

(b) Internal causes

1.3.1. External causes:-

* External transients are sudden change in voltages occur due to lightning

Lightning:-

* An electric discharge between cloud and Earth, between clouds or between the charge centres of the same cloud is 'Lightning'

* It produces extremely powerful, short duration transients on power distribution systems either by a direct strike or a near hit

* In most cases, a lightning strike induced surge on local power distribution line causes damage to susceptible Equipment.

* The overvoltages created by these transients will be ~~cleared~~ clamped by lightning arresters to a level the substation equipment can handle without any damages to it.

Secondary Effects:-

It also causes problems to

(i) Electronic equipment

(ii) Computers

(iii) SMPS

(iv) convertors etc.,

Other External causes:-

(i) Poor (or) Loose Connections in the distribution system

(ii) High wind which blows one Power line ~~to~~ into another

(iii) Accidents and human Error

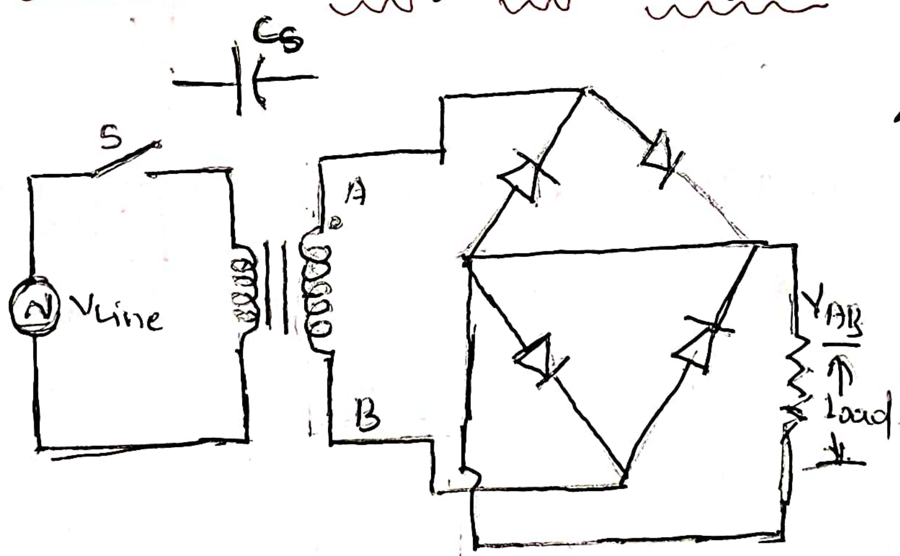
1.3.2. Internal causes:-

* Internal transients are sudden change in voltage occur due to opening and closing of switches.
* Transients produced due to this may increase the system voltage to twice the normal voltage.
* By providing proper insulation internal causes can be reduced.

Example of Internal causes:-

(a) switch opening (b) Transformer primary being Energized
(c) Transformer primary being de-energized
(d) capacitor switching
(e) Line Energizing etc.,

(i) Transformer primary being Energized:-



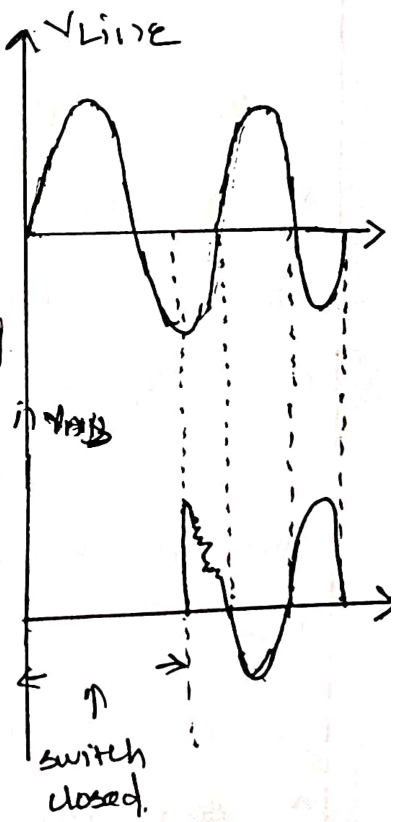


Fig:- Transformer capacitance causes a transient

* If the transformer is energized at the peak of the line voltage, then this voltage can couple to the stray capacitance and inductances of the secondary winding

* This generates the oscillating transient voltage

* This transient peak voltage can be upto twice the peak amplitude of normal secondary voltage.

(i) Capacitance Switching:-

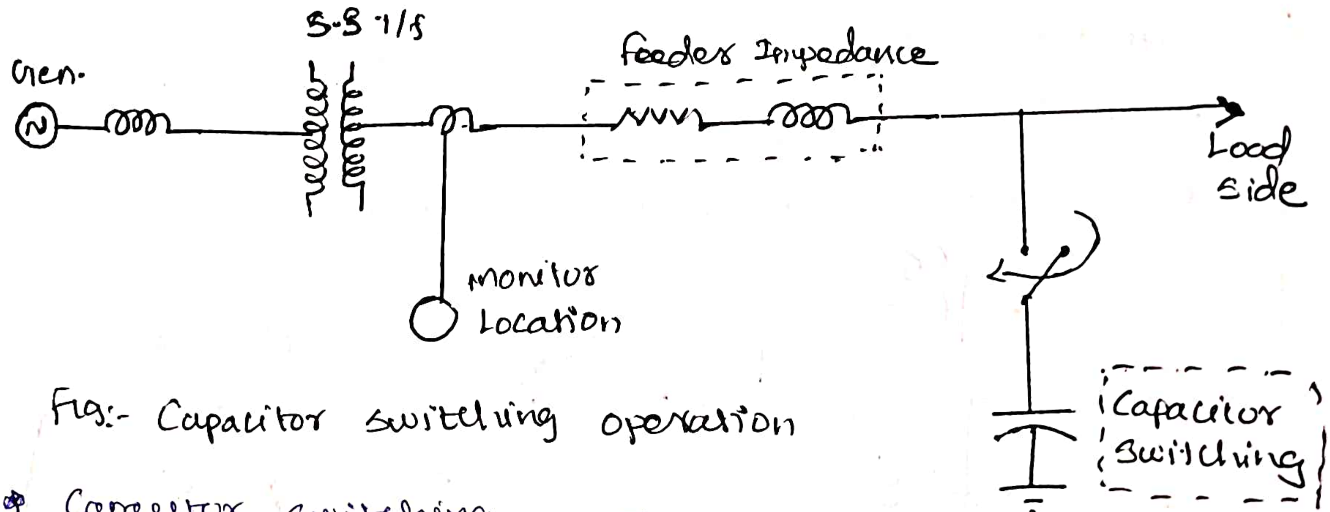


Fig:- Capacitor switching operation

* Capacitor switching is the most common switching events on utility side and it creates transients with oscillatory type of waveform.

* This type of transient can propagate into the utility's side power system, enter into the end-users load through its corresponding distribution transformers.

* It will damage the user's equipment.

* So it has to be prevented before making any adverse effect.

Wave form :- (capacitor switching)

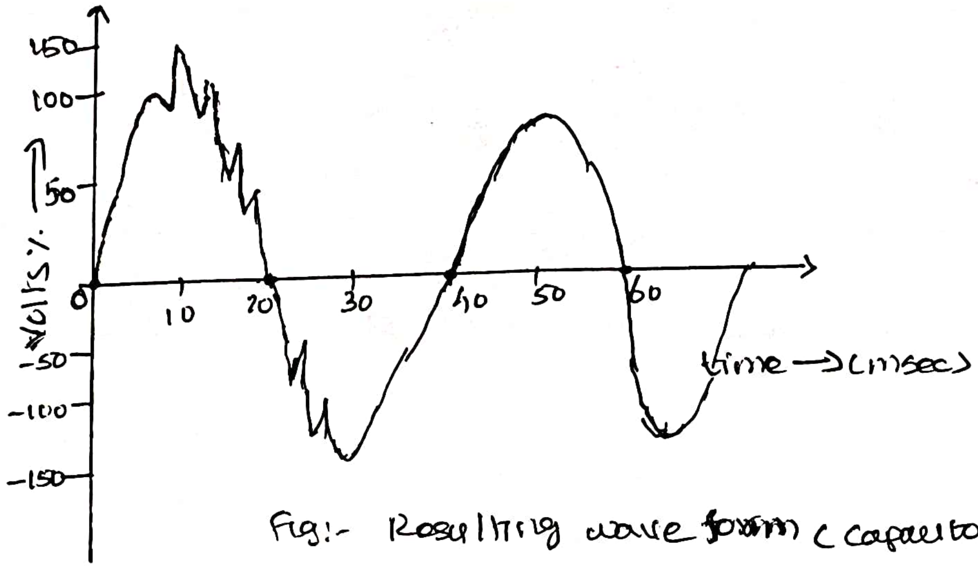


Fig:- Resulting wave form (capacitor switching)

1.4 R-L Transient with sine wave Excitation:-

There are two problems mainly considered for ~~analyzing~~ analizing the transient in the circuit.

(i) The closing of switch (or) circuit Breaker to Energize a load (sine wave Excitation)

(ii) Opening of Breaker to clear a fault (Double frequency Excitation).

(i) Closing of switch:- (sine wave Excitation)

Transient is initiated whenever there is sudden change ~~of~~ of circuit conditions. This most frequently occurs when a switching operations takes place.

The circuit involved is shown in fig.

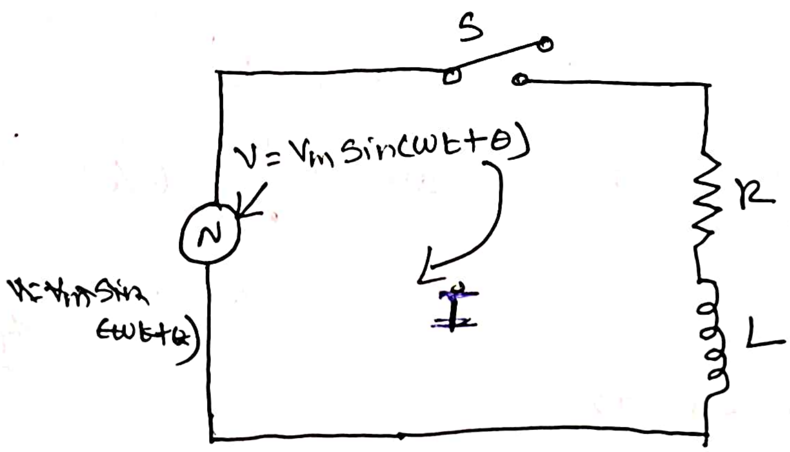


Fig. R-L circuit with a sinusoidal drive.

* When switch 'S' is closed then the equation (Kirchoff's voltage law)

$$\boxed{RI + L\frac{dI}{dt} = V_m \sin(\omega t+\theta)} \quad \text{--- (1)}$$

Divide with 'L' on both sides of equation no 1

$$\frac{RI}{L} + \frac{dI}{dt} = \frac{V_m}{L} \sin(\omega t + \theta)$$

$$I\left[\frac{d}{dt} + \frac{R}{L}\right] = \frac{V_m}{L} \sin(\omega t + \theta) \qquad \left[\because \frac{d}{dt} = D\right]$$

$$\boxed{\left[D + \frac{R}{L}\right] I = \frac{V_m}{L} \sin(\omega t + \theta)} \quad \text{—— } (2)$$

* It is non-homogeneous equation, it contains two terms complimenery function & particular integral.

$$\boxed{\therefore \quad I = I_c + I_p \text{ —— } (3)}$$

$$\boxed{I_c = c e^{-t\left(\frac{R}{L}\right)} \text{ —— } (4)}$$

$$\boxed{I_p = A \cos(\omega t + \theta) + B \sin(\omega t + \theta)} \text{ —— } (5)$$

~~To find constants A & B~~

Differentiate equation no.(5) w.r.to 't'

$$\boxed{I_p' = -A\omega \sin(\omega t + \theta) + B\omega \cos(\omega t + \theta)} \text{ —— } (6)$$

* To find constants A & B sub (5) & (6) in eqn no:(2) (7)

Assume I as I_p in eqn (2)

$$\left(D + \frac{R}{L}\right) I_p = \frac{V_m}{L} \sin(\omega t + \theta)$$

$$D I_p + \frac{R}{L} I_p = \frac{V_m}{L} \sin(\omega t + \theta)$$

$$\boxed{I_p' + \frac{R}{L} I_p = \frac{V_m}{L} \sin(\omega t + \theta)} \quad \text{--- (7)}$$

$\left[D = \frac{d}{dt} \right.$

$\therefore D I_p = \frac{d I_p}{dt}$

$\left. = I_p' \right]$

$$\left[\left[-A\omega \sin(\omega t + \theta) + B\omega \cos(\omega t + \theta) \right] + \frac{R}{L} \left[A \cos(\omega t + \theta) + B \sin(\omega t + \theta) \right] \right.$$

$$\left. = \frac{V_m}{L} \sin(\omega t + \theta) \right] \quad \text{--- (8)}$$

Rearrange eqn no. 8)

$$\left[\sin(\omega t + \theta) \left[-A\omega + \frac{BR}{L} \right] + \cos(\omega t + \theta) \left[B\omega + \frac{RA}{L} \right] \right.$$

$$\left. = \frac{V_m}{L} \sin(\omega t + \theta) \right] \quad \text{--- (9)}$$

compare sin. & cos terms in equation no. (9)

$\Rightarrow$ (9)

$$\left[-A\omega + \frac{BR}{L} \right] = \frac{V_m}{L} \quad \text{--- (10)}$$

$$B\omega + \frac{RA}{L} = 0$$

$$B\omega = -\frac{RA}{L}$$

$$\boxed{B = -\frac{RA}{\omega L}} \quad \text{--- (11)}$$

sub (11) in (10)

$$\cancel{-A\omega + \left[\frac{-RA}{\omega L} \right] = \frac{V_m}{L}} \quad \left[-A\omega + \cdot -\frac{R^2 A}{\omega L^2} \right] = \frac{V_m}{L}$$

$$A\left[-\omega - \frac{R^2}{\omega L^2}\right] = \frac{V_m}{L}$$

$$A\left[\frac{-\omega^2 L^2 - R^2}{\omega L^2}\right] = \frac{V_m}{L}$$

$$-A\left[\frac{\omega^2 L^2 + R^2}{\omega L^2}\right] = \frac{V_m}{L}$$

$$-A = \frac{V_m\, \omega L^{\cancel{2}}}{\cancel{L}\left[R^2 + \omega^2 L^2\right]}$$

sub (12) in (11)

$$\boxed{A = \frac{-V_m \omega L}{\left[R^2 + \omega^2 L^2\right]}} \quad \text{——— (12)}$$

$$\therefore \boxed{B = \frac{V_m R}{R^2 + (\omega L)^2}} \quad \text{——— (13)}$$

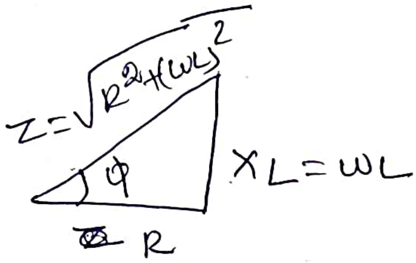
sub (12) & (13) in equation no. (5)

$$(5) \Rightarrow I_p = -\frac{V_m \omega L}{R^2 + (\omega L)^2}\cos(\omega t + \theta) + \frac{V_m R}{R^2 + (\omega L)^2}\sin(\omega t + \theta) \quad \text{——— (14)}$$

W.K.T

$$\left.\begin{aligned} \sin\phi &= \frac{\omega L}{\sqrt{R^2 + (\omega L)^2}} \\ \cos\phi &= \frac{R}{\sqrt{R^2 + (\omega L)^2}} \\ \tan\phi &= \frac{\omega L}{R} \end{aligned}\right\} \text{— (15)}$$

[From Impedance triangle



$$(14) \Rightarrow \frac{-V_m\,\omega L}{\sqrt{R^2+(\omega L)^2}\cdot\sqrt{R^2+(\omega L)^2}}\cos(\omega t+\theta) + \frac{V_m\,R}{\sqrt{R^2+(\omega L)^2}\cdot\sqrt{R^2+(\omega L)^2}}\cdot\sin(\omega t+\theta)$$

$$\left[R^2+(\omega L)^2 = \sqrt{R^2+(\omega L)^2}\cdot\sqrt{R^2+(\omega L)^2}\right]$$

sub (15) in (14)

Now

$$\Rightarrow I_p = \frac{-V_m}{\sqrt{R^2+(\omega L)^2}}\cdot\sin\phi\,\cos(\omega t+\theta) + \frac{V_m}{\sqrt{R^2+(\omega L)^2}}\cos\phi\,\sin(\omega t+\theta)$$

$$I_p = \frac{V_m}{\sqrt{R^2+(\omega L)^2}}\sin(\omega t+\theta)\cdot\cos\phi - \frac{V_m}{\sqrt{R^2+(\omega L)^2}}\cos(\omega t+\theta)\cdot\sin\phi$$

$$\left[\because \sin(A-B) = \sin A\cos B - \cos A\sin B\right.$$

$$A = \omega t+\theta,\quad B=\phi$$

$$\Rightarrow I_p = \frac{V_m}{\sqrt{R^2+(\omega L)^2}}\sin\left((\omega t+\theta)-\phi\right) \qquad \left[\phi = \tan^{-1}\frac{\omega L}{R}\right]$$

$$\boxed{I_p = \frac{V_m}{\sqrt{R^2+(\omega L)^2}}\sin(\omega t+\theta-\phi)} \quad \text{——— (16)}$$

$$\boxed{I = I_c + I_p} \quad \text{——— (17)}$$

sub (4) & (16) in equation no. 17

$$\boxed{I = c\,e^{-\frac{R}{L}t} + \frac{V_m}{\sqrt{R^2+(\omega L)^2}}\sin(\omega t+\theta-\phi)} \quad \text{——— (18)}$$

To find the value of constant 'c' Apply boundary condition

$\boxed{t=0 \ \& \ I=0}$ in eqn no. 18

$$0 = ce^{-\frac{R}{L}\times 0} + \frac{V_m}{\sqrt{R^2+(\omega L)^2}} \sin(\omega\times 0+\theta-\phi)$$

$$0 = c + \frac{V_m}{\sqrt{R^2+(\omega L)^2}} \sin(\theta-\phi)$$

$$\boxed{c = \frac{-V_m}{\sqrt{R^2+(\omega L)^2}} \sin(\theta-\phi)} \quad \text{--- } (19)$$

sub (19) in (18)

$$I = e^{-\frac{R}{L}t}\left[\frac{-V_m}{\sqrt{R^2+(\omega L)^2}} \sin(\theta-\phi)\right] + \frac{V_m}{\sqrt{R^2+(\omega L)^2}} \sin(\omega t+\theta-\phi)$$

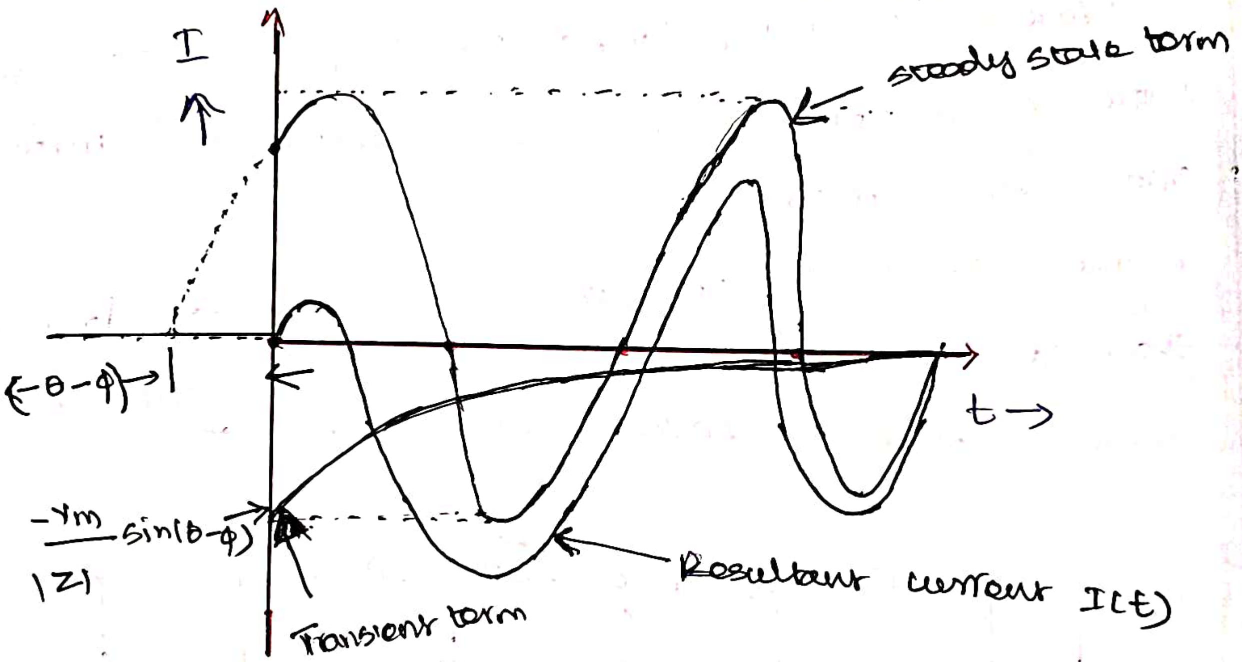
$$\boxed{I = \frac{V_m}{|Z|}\left[\sin(\omega t+\theta-\phi)\right] - e^{-\frac{R}{L}t}\cdot\frac{V_m}{|Z|}\left[\sin(\theta-\phi)\right]} \quad \text{--- } (20)$$

From the above equation the first term is Steady state final value. Its amplitude is $\frac{V_m}{|Z|}$ and it indeed has a phase angle $\boxed{-\phi}$ with respect to voltage

* The second term is transient. It involves as exponential $e^{-\frac{R}{L}t}$, moreover at $\boxed{t=0}$ it is equal and opposite to the steady state term thus assuming that current starts from zero

Case i when the switch 'S' closes at instant $\boxed{\theta=\phi}$

* The transient term will become zero in (eqn no. 20) and current wave will be symmetrical.



Case ii:- If the switch (S) closes at instant $\theta-\phi=\pm\frac{\pi}{2}$

→ The transient term attains its maximum value

→ First peak of the resultant component resulting composite current wave will approach twice the peak amplitude of the steady state sinusoidal component.

1.5 DOUBLE FREQUENCY RECOVERY TRANSIENTS

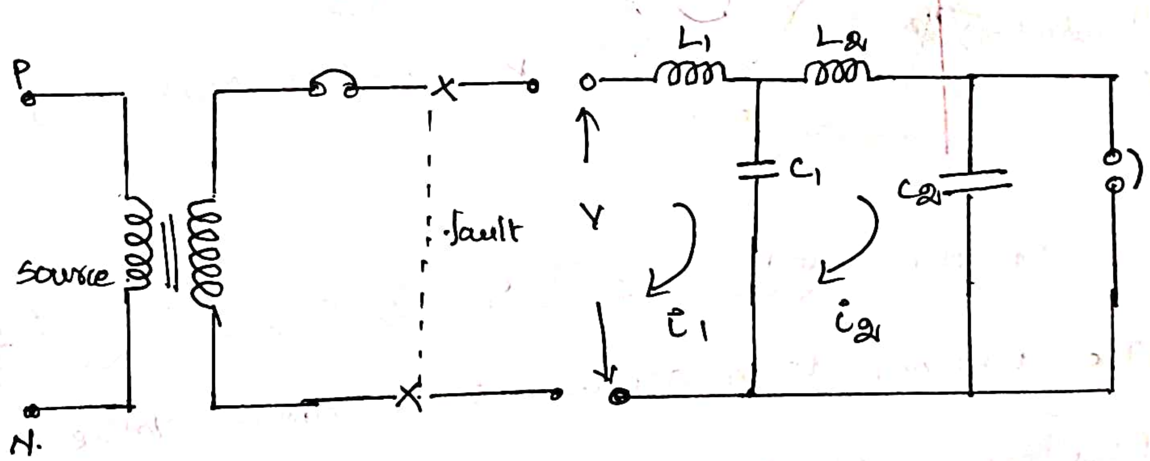
(Circuit Breakers clearing the fault)

* Double frequency transients occurs after the fault is cleared. ie the arc has been extinguished, both the circuits oscillates at their own natural frequencies and a composite double frequency transients appears across the circuit breaker pole. (eg:- Unloaded T/f)

Double frequency circuits:-

* There are many double frequency circuits are there in practice. One case is encountered quite and often shown below.

* This shows a ~~circu~~ circuit breaker ~~clea~~ clearing a short circuit on the secondary side of transformer



Where $L_1 \rightarrow$ Transformer inductance

$L_2 \rightarrow$ Transformer leakage inductance

$C_1, C_2 \rightarrow$ Inherent system capacitance across transformer side.

Loop 1:- Apply KVL

sum of Rise in voltage = sum of drop in voltage

i.e
$$V = V_{L_1} + V_{C_1} \quad \text{—— (1)}$$

$$V_{C_1} = V - V_{L_1}$$

$$V_{C_1} = V - L_1 \frac{dI_1}{dt} \quad \text{—— (2)}$$

$$\frac{1}{C_1}\int (I_1 - I_2)\, dt + V_{C_1}(0) = V - L_1 \frac{dI_1}{dt} \quad \text{—— (3)}$$

Apply Laplace Transform on both sides:- of eqn (3)

$$\frac{V}{S} + L_1[S I_1(S) - I_1(0)] = \frac{I_1(S)}{C_1}$$

$$\frac{V}{S} - L_1 S I_1(S) + L_1 I_1(0) = \frac{1}{C_1 S}[I_1(S) - I_2(S)] + \frac{V_{C_1}(0)}{S}$$

$$\boxed{\frac{V}{S} - L_1[S I_1(S) - I_1(0)] = \frac{I_1(S)}{C_1 S} - \frac{I_2(S)}{C_1 S} + \frac{V_{C_1}(0)}{S}} \quad \text{—— (4)}$$

Apply boundary condition $\boxed{I_1(0) = 0}$ @ eqn no.(4)

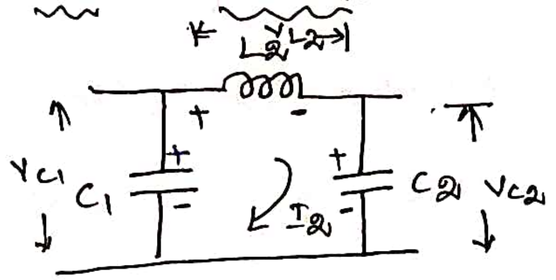
$$\frac{V}{S} - L_1 S I_1(S) + 0 = \frac{I_1(S)}{C_1 S} - \frac{I_2(S)}{C_1 S} + \frac{V_{C_1}(0)}{S}$$

Take $I_1(S)$ as common

$$\frac{I_1(S)}{C_1 S} + L_1 S I_1(S) = \frac{V}{S} + \frac{I_2(S)}{C_1 S} - \frac{V_{C_1}(0)}{S}$$

$$\boxed{I_1(S)\left[L_1 S + \frac{1}{C_1 S}\right] - \frac{I_2(S)}{C_1 S} = \frac{V}{S} - \frac{V_{C_1}(0)}{S}} \quad \text{—— (5)}$$

Apply KVL @ Loop 2:-



[rise in P↑ = Drop in P↓
ie (− to +) → rise
(+ to −) → drop]

$$V_{C1} = V_{L2} + V_{C2} \longrightarrow (6)$$

$$V_{C1} = L_2 \cdot \frac{di_2}{dt} + V_{C2}$$

$$V_{C2} = V_{C1} - L_2 \cdot \frac{di_2}{dt} \quad \text{—— (7)}$$

sub (2) in (7)

$$V_{C2} = V - L_1 \frac{di_1}{dt} - L_2 \cdot \frac{di_2}{dt} \quad \text{—— (8)}$$

Apply Laplace Transform on both sides of equation number (8)

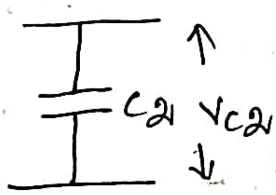
$$V_{C2}(s) = \frac{V}{s} - L_1 [s I_1(s) - s I_1(0)] - L_2 [s I_2(s) - I_2(0)] \quad \text{—— (9)}$$

Apply Boundary Condition @ equation no. (9)

$$\boxed{V_{C2}(s) = \frac{V}{s} - L_1 s I_1(s) - L_2 s I_2(s)} \quad \text{—— (10)}$$

Now voltage across the capacitor 'C_2' [for simplifying the equation]

ie



$$\boxed{V_{C2} = \frac{1}{C_2} \int i_2 \cdot dt} \quad \text{—— (11)}$$

(10)

Apply Laplace Transform on both sides of equation no.(11)

$$V_{C2}(S) = \frac{I_2(S)}{C_2 S}$$

$$\boxed{I_2(S) = V_{C2}(S) \cdot C_2 S} \quad \text{—— (12)}$$

Sub. (12) in equation no.(10)

$$V_{C2}(S) = \frac{V}{S} - L_1 S\, I_1(S) - L_2 S^2 V_{C2}(S) \cdot C_2$$

Re arranging the above equation

$$V_{C2}(S) + L_2 S^2 V_{C2}(S) \cdot C_2 = \frac{V}{S} - L_1 S\, I_1(S)$$

$$\cancel{L_1 S}\; V_{C2}(S)\left[1 + L_2 S^2 C_2\right] = \frac{V}{S} - L_1 S\, I_1(S)$$

$$L_1 S\, I_1(S) = \frac{V}{S} - V_{C2}(S)\left[1 + L_2 S^2 C_2\right]$$

$$\boxed{I_1(S) = \frac{V}{S^2 L_1} - \frac{V_{C2}(S)}{L_1 S}\left[1 + L_2 S^2 C_2\right]} \quad \text{—— (13)}$$

Sub (12) & (13) in equation no.(5)

$$\boxed{\left[\frac{V}{L_1 S^2} - \frac{V_{C2}(S)}{L_1 S}\left[1 + L_2 C_2 S^2\right]\right]\left[L_1 S + \frac{1}{C_1 S}\right] - \left[\frac{C_2 \cancel{S}\, V_{C2}(S)}{\cancel{S} C_1}\right] = \frac{V}{S} - \frac{V_{C1}(0)}{S}} \quad \text{—— (14)}$$

× above eqn by (− sign)

$$\cancel{\frac{x}{L_1 S^2}}\left[\frac{V_{C2}(S)}{L_1 S}\left[1 + L_2 C_2 S^2\right] - \frac{V}{L_1 S^2}\right]\left[L_1 S + \frac{1}{C_1 S}\right] + \frac{C_2}{C_1} V_{C2}(S)$$

$$= \frac{V_{C1}(0)}{S} - \frac{V}{S}$$

$$\left[\left[\frac{V_{C2}(s)}{\cancel{L_1 s}} \times \cancel{L_1 s} + \frac{V_{C2}(s)}{L_1 C_1 s^2}\right]\left[1 + L_2 C_2 s^2\right]\right] - \left[\frac{V}{L_1 s^2}\right]\left[L_1 s + \frac{1}{s C_1}\right]$$

$$+ \frac{C_2}{C_1} V_{C2}(s) = \frac{V_{C1}(0)}{s} - \frac{V}{s}$$

$$V_{C2}(s) + V_{C2}(s) L_2 C_2 s^2 + \frac{V_{C2}(s)}{L_1 C_1 s^2} + \frac{V_{C2}(s)}{L_1 C_1 \cancel{s^2}} \times L_2 C_2 \cancel{s^2} - \frac{V}{\cancel{L_1} s^{\cancel{2}}} \times \cancel{L_1 s} - \frac{V}{L_1 C_1 s^3}$$

$$+ \frac{C_2}{C_1} V_{C2}(s) = \frac{V_{C1}(0)}{s} - \frac{V}{s}$$

$$\boxed{V_{C2}(s) + V_{C2}(s) L_2 C_2 s^2 + \frac{V_{C2}(s)}{L_1 C_1 s^2} + \frac{V_{C2}(s)}{L_1 C_1} \times L_2 C_2 - \frac{V}{s} - \frac{V}{L_1 C_1 s^3} + \frac{C_2}{C_1} V_{C2}(s) = \frac{V_{C1}(0)}{s} - \frac{V}{s}} \rightarrow (15)$$

xly the above equation by $\frac{s^2}{L_2 C_2}$ on both sides

$$V_{C2}(s) \cdot \frac{s^2}{L_2 C_2} + V_{C2}(s) \cancel{L_2 C_2} s^2 \times \frac{s^2}{\cancel{L_2 C_2}} + \frac{V_{C2}(s)}{L_1 C_1 \cancel{s^2}} \times \frac{\cancel{s^2}}{L_2 C_2} + \left[\frac{V_{C2}(s)}{L_1 C_1} \times L_2 C_2\right] \times \frac{s^2}{L_2 C_2}$$

$$-\left[\frac{V}{\cancel{s}}\right]\left[\frac{s^{\cancel{2}}}{L_2 C_2}\right] - \left[\frac{V}{L_1 C_1 s^{\cancel{3}}}\right] \times \frac{s^2}{L_2 C_2} + \left[\frac{\cancel{C_2}}{C_1} V_{C2}(s)\right] \times \frac{s^2}{L_2 \cancel{C_2}}$$

$$= \frac{V_{C1}(0)}{\cancel{s}} \times \frac{s^{\cancel{2}}}{L_2 C_2} - \left[\frac{V}{\cancel{s}}\right] \times \left[\frac{s^{\cancel{2}}}{L_2 C_2}\right] \rightarrow (16)$$

$$\boxed{V_{C2}(s) \cdot \frac{s^2}{L_2 C_2} + V_{C2}(s) \cdot s^4 + \frac{V_{C2}(s)}{L_1 C_1 L_2 C_2} + \frac{V_{C2}(s)\, s^2}{L_1 C_1} - \frac{V \cdot s}{L_2 C_2} - \frac{V}{L_1 C_1 L_2 C_2 s} + V_{C2}(s) \cdot \frac{s^2}{L_2 C_1} = \frac{V_{C1}(0) \cdot s}{L_2 C_2} - \frac{V s}{L_2 C_2}}$$

Rearranging equation no (16)

$$V_{c2}(s)\left[s^4+s^2\left[\frac{1}{L_1C_1}+\frac{1}{L_2C_2}+\frac{1}{L_2C_1}\right]+\frac{1}{L_1C_1L_2C_2}\right]$$

$$=\frac{V_{c1}(0)\,s}{L_2C_2}-\cancel{\frac{V_g}{L_2C_2}}+\cancel{\frac{V_g}{L_2C_2}}+\frac{V}{L_1C_1L_2C_2S}$$

$$\boxed{V_{c2}(s)\left[s^4+s^2\left[\frac{1}{L_1C_1}+\frac{1}{L_2C_2}+\frac{1}{L_2C_1}\right]+\frac{1}{L_1C_1L_2C_2}\right]=\frac{V_{c1}(0)\,s}{L_2C_2}+\frac{V}{L_1C_1L_2C_2S}}\rightarrow(17)$$

W.K.T

voltage across the capacitor

i.e $V_{c1}(0)=\left[\frac{L_2}{L_1+L_2}\right]\cdot V$ —— (18) [voltage division rule refer ~~cir~~circuit]

Sub (18) in (17)

$$V_{c2}(s)\left[s^4+s^2\left[\frac{1}{L_1C_1}+\frac{1}{L_2C_2}+\frac{1}{L_2C_1}\right]+\frac{1}{L_1C_1L_2C_2}\right]$$

$$=\left[\frac{\cancel{L_2}}{L_1+L_2}\right]\cdot\frac{V\,s}{\cancel{L_2}C_2}+\frac{V}{L_1C_1L_2C_2\,S}$$

$$\boxed{V_{c2}(s)\left[s^4+s^2\left[\frac{1}{L_1C_1}+\frac{1}{L_2C_2}+\frac{1}{L_2C_1}\right]+\frac{1}{L_1C_1L_2C_2}\right]=V\left[\frac{s}{(L_1+L_2)C_2}+\frac{1}{L_1C_1L_2C_2\,s}\right]}$$ —— ~~(18)~~ (19)

The expression on the left hand side of eqn (19) quadratic in s^2. Therefore the equation can be rewritten as

$$\boxed{s^4 + s^2\left[\frac{1}{L_1C_1} + \frac{1}{L_2C_2} + \frac{1}{L_2C_1}\right] + \frac{1}{L_1C_1\,L_2C_2} = 0} \quad \text{— (20)}$$

Let the roots be ω_1^2 & ω_2^2 i.e $\left[\omega_1 = \frac{1}{L_1C_1}, \quad \omega_2 = \frac{1}{L_2C_2}\right]$

∴ Equation (20) can be written as

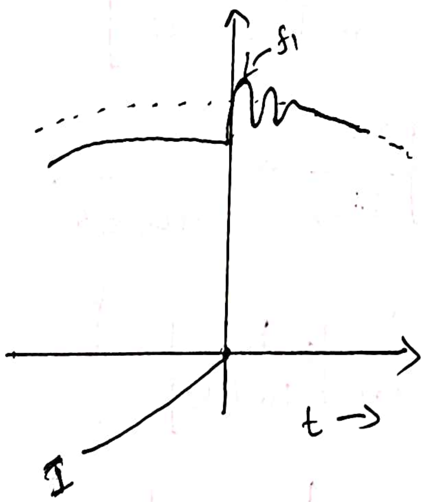
$$(s^2+\omega_1^2)(s^2+\omega_2^2) = 0$$

Equation (19) becomes,

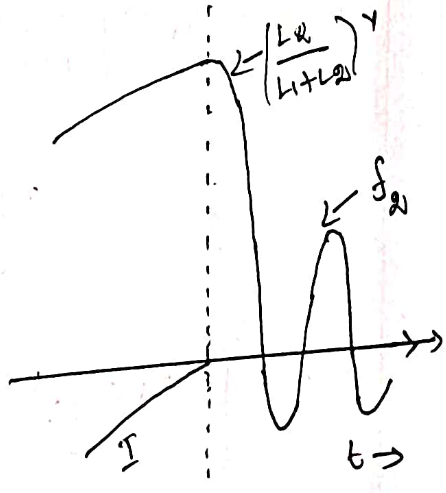
$$\boxed{V_{C2}(s)\left[(s^2+\omega_1^2)(s^2+\omega_2^2)\right] = V\left[\frac{A}{s} + Bs\right]} \quad \text{— (21)}$$

where $\left\{ A = \frac{1}{L_1C_1\,L_2C_2} \quad B = \frac{1}{(L_1+L_2)C_2} \right\}$ — (22)

Equation no. 20 wave forms:-



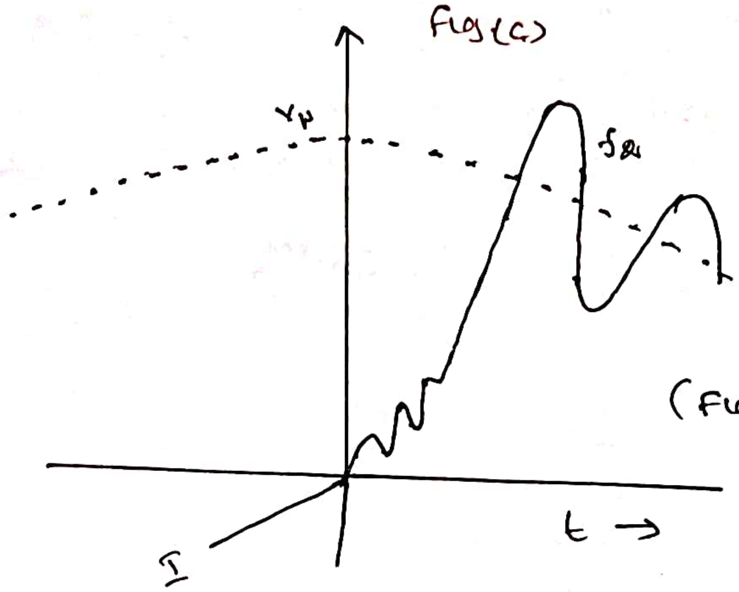
(a)



(b)

(a) Source side transient (b) Load side transient

Fig(c)



(Fig.c)

Fig(c). Recovery voltage across the switch

* Equation no (21) shows the circuit breaker recovery voltage obtained in which A, B are constants, depends upon the circuit parameter $\boxed{L_1, L_2, C_1, C_2}$

—x —x —x —x

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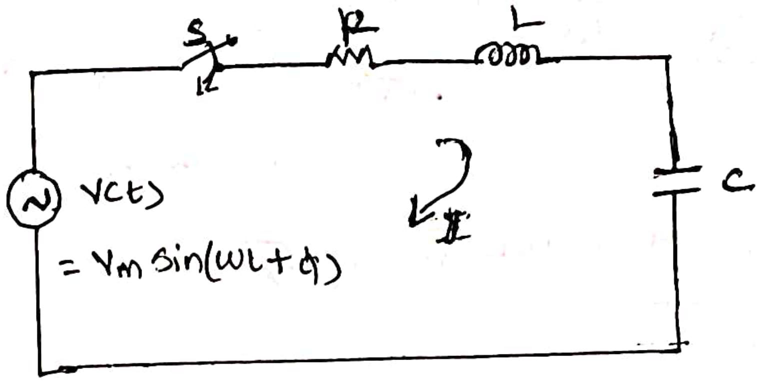
1.6 BASIC TRANSFORM OF SERIES RLC CIRCUIT TRANSIENTS (13) (26)

* Consider a simple series R-L-C circuit with sinusoidal Input as shown in fig.



* Applying KVL to the circuit we get the first order differential equation as

$$\boxed{RI + L\frac{dI}{dt} + \frac{1}{C}\int I \cdot dt = v(t)} \quad \text{—— (1)}$$

* Diff eqn (1) w.r.to 't' :-

$$\boxed{R \cdot \frac{dI}{dt} + L\frac{d^2I}{dt^2} + \frac{1}{C} I = \frac{dv(t)}{dt}} \quad \text{—— (A)}$$

Re arranging the above equation (2)

$$L \cdot \frac{d^2I}{dt^2} + R \cdot \frac{dI}{dt} + \frac{1}{C} I = \frac{dv(t)}{dt}$$

÷ by L

$$\boxed{\frac{d^2I}{dt^2} + \frac{R}{L} \cdot \frac{dI}{dt} + \frac{1}{LC} I = \frac{dv(t)}{dt} \cdot \frac{1}{L}} \quad \text{—— (2)}$$

* The solution of equation (2) which will have two parts namely complementary function and particular integral gives the total current.

i.e. $\boxed{I = I_c + I_p}$ ——— (3) $\boxed{\left[D^2 + \frac{R}{L}D + \frac{1}{LC}\right] I = \frac{D V_m}{L}}$ (4)

The characteristic equation of the system is

$$\boxed{\left[D^2 + \frac{R}{L}D + \frac{1}{LC}\right] I = 0} \quad (5)$$

[where $D^2 = \frac{d^2}{dt^2}$

$D = \frac{d}{dt}$

Excitation $V_m = 0$]

To find I_c:-

∴ The solution of equation no:- 5

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

(4) ⇒ $\boxed{b = \frac{R}{L}; \quad a = 1, \; c = \frac{1}{LC}}$

$$\therefore \lambda = \frac{-\frac{R}{L} \pm \sqrt{\left[\frac{R}{L}\right]^2 - 4 \times 1 \times \frac{1}{LC}}}{2 \times 1}$$

[Bring '2' inside root]

$\sqrt{\frac{1}{4}} = \frac{1}{2}$

$$\lambda = \frac{-R}{2L} \pm \sqrt{\frac{1}{4}\left[\left[\frac{R}{L}\right]^2 - \frac{4}{LC}\right]}$$

$$\lambda = \frac{-R}{2L} \pm \sqrt{\left[\frac{R}{2L}\right]^2 - \frac{1}{LC}}$$

$$\boxed{\begin{aligned} \therefore \lambda_1 &= \frac{-R}{2L} + \sqrt{\left[\frac{R}{2L}\right]^2 - \frac{1}{LC}} \\ \lambda_2 &= \frac{-R}{2L} - \sqrt{\left[\frac{R}{2L}\right]^2 - \frac{1}{LC}} \end{aligned}} \quad (6)$$

$$\boxed{\therefore\ I_c = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t}} \quad \text{--- (7)}$$

where $c_1, c_2 \rightarrow$ orbitary constant of the circuit.

Equation (7) {Transient} {component} of the total current in the circuit.

To find particular Integral:- (I_p)

* Generalized current equation

$$\boxed{I_p = A\cos(\omega t+\phi) + B\sin(\omega t+\phi)} \quad \text{--- (8)}$$

Sub (8) in equation no. (4)

$$(4) \Rightarrow \left[D^2 + \frac{R}{L}\cdot D + \frac{1}{LC}\right] I_p = D\frac{V_m}{L} \quad \left[\because \text{Let } I \text{ as } I_p \text{ in (4)}\right]$$

$$\left[D^2 + \frac{R}{L}\cdot D + \frac{1}{LC}\right]\left[A\cos(\omega t+\phi) + B\sin(\omega t+\phi)\right] = D\left[\frac{V_m}{L}\right]$$

ie $$D^2\left[A\cos(\omega t+\phi) + B\sin(\omega t+\phi)\right] + \frac{R}{L}\cdot D\left[A\cos(\omega t+\phi) + B\sin(\omega t+\phi)\right]$$

$$+\frac{1}{LC}\left[A\cos(\omega t+\phi) + B\sin(\omega t+\phi)\right] = \frac{d}{dt}\left[\frac{V_m\sin(\omega t+\phi)}{L}\right] \quad \text{--- (9)}$$

[Note :- 'D' means differentiation] $\left[\because D = \frac{d}{dt}\right]$

$$(9) \Rightarrow \left[-A\omega^2\cos(\omega t+\phi) - B\omega^2\sin(\omega t+\phi)\right] + \frac{R}{L}\left[-A\omega\sin(\omega t+\phi) + B\omega\cos(\omega t+\phi)\right]$$

$$+\frac{1}{LC}\left[A\cos(\omega t+\phi) + B\sin(\omega t+\phi)\right] = \frac{V_m\omega\cos(\omega t+\phi)}{L}$$

Rearranging the above equation we get

$$\Big[-A\omega^2\cos(\omega t+\phi)+\frac{B\omega R}{L}\cos(\omega t+\phi)+\frac{A}{LC}\cos(\omega t+\phi)$$

$$-B\omega^2\sin(\omega t+\phi)-\frac{A\omega R}{L}\sin(\omega t+\phi)+\frac{B}{LC}\sin(\omega t+\phi)\Big]=$$

$$=\frac{V_m\omega}{L}\cos(\omega t+\phi)$$

$$\boxed{\cos(\omega t+\phi)\left[-A\omega^2+\frac{B\omega R}{L}+\frac{A}{LC}\right]+\sin(\omega t+\phi)\left[-B\omega^2-\frac{AR\omega}{L}+\frac{B}{LC}\right]=\frac{V_m\omega}{L}\cos(\omega t+\phi)} \quad (10)$$

comparing (cos) & (sin) terms on both sides of eqn no(10)

$$-A\omega^2+\frac{B\omega R}{L}+\frac{A}{LC}=\frac{V_m\omega}{L}$$

$$\boxed{A\left[\frac{1}{LC}-\omega^2\right]+\frac{B\omega R}{L}=\frac{V_m\omega}{L}} \quad (11)$$

$$-B\omega^2-\frac{AR\omega}{L}+\frac{B}{LC}=0$$

$$\boxed{B\left[\frac{1}{LC}-\omega^2\right]-\frac{AR\omega}{L}=0} \quad (12)$$

$$(12)\Rightarrow B\left[\frac{1}{LC}-\omega^2\right]=\frac{AR\omega}{L}$$

$$B=\frac{AR\omega}{L\left[\frac{1}{LC}-\omega^2\right]}$$

15 (b)

$$B = \frac{ARW}{L\left[\frac{1-\omega^2 LC}{LC}\right]}$$

$$B = \frac{AR\omega \times \cancel{L}C}{\cancel{L}\left[1-\omega^2 LC\right]}$$

$$B = \frac{AR\cancel{\omega}\cancel{C}}{\cancel{\omega}\cancel{C}\left[\frac{1}{\omega C} - \omega L\right]}$$

$$\boxed{B = \frac{AR}{\left[\frac{1}{\omega C} - \omega L\right]}} \longleftarrow (13)$$

sub(13) in equation (11)

$$(11) \Rightarrow A\left[\frac{1}{LC} - \omega^2\right] + \frac{B\omega R}{L} = \frac{V_m\omega}{L}$$

$$A\left[\frac{1}{LC} - \omega^2\right] + \frac{A\ R^2\omega}{L\left[\frac{1}{\omega C} - \omega L\right]} = \frac{V_m\omega}{L}$$

$$A \times \frac{\omega}{L\left[\frac{1}{\omega C} - \omega L\right]}\left[R^2 + \left[\frac{1}{LC} - \omega^2\right] \times \frac{L\left[\frac{1}{\omega C} - \omega L\right]}{\omega}\right] = \frac{V_m\omega}{L}$$

$$A\left[R^2 + \left[\frac{1}{\cancel{L}C} \times \frac{\cancel{L}}{\omega} - \omega^2 \times \frac{L}{\cancel{\omega}}\right] \times \left[\frac{1}{\omega C} - \omega L\right]\right] = \frac{V_m\cancel{\omega}}{\cancel{L}} \times \frac{L\left[\frac{1}{\omega C} - \omega L\right]}{\omega}$$

$$A\left[R^2+\left[\frac{1}{\omega C}-\omega L\right]\left[\frac{1}{\omega C}-\omega L\right]\right] = V\left[\frac{1}{\omega C}-L\omega\right]$$

$$A\left[R^2+\left[\frac{1}{\omega C}-\omega L\right]^2\right] = V\left[\frac{1}{\omega C}-\omega L\right]$$

$$\boxed{A = \frac{V\left[\frac{1}{\omega C}-\omega L\right]}{R^2+\left[\frac{1}{\omega C}-\omega L\right]^2}} \quad \text{--- (14)}$$

sub (14) in (13)

$$B = \frac{V\left[\frac{1}{\omega C}-\cancel{\omega L}\right]}{R^2+\left[\frac{1}{\omega C}-\omega L\right]^2} \times \frac{R}{\left[\frac{1}{\omega C}\cancel{-\omega L}\right]}$$

$$\boxed{B = \frac{VR}{R^2+\left[\frac{1}{\omega C}-\omega L\right]^2}} \quad \text{--- (15)}$$

sub (14) & (15) in equation (8)

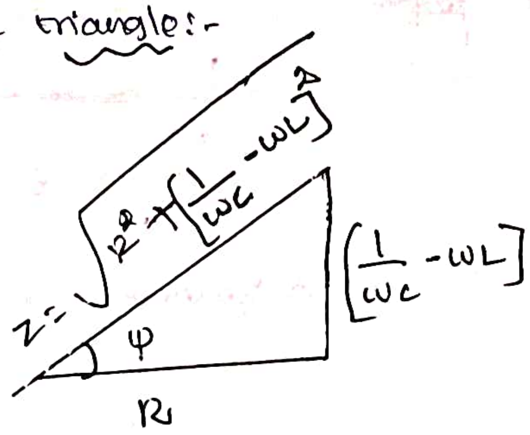
(8) $\Rightarrow$ $I_p = A\cos(\omega t+\phi) + B\sin(\omega t+\phi)$

$$\boxed{I_p = \frac{V\left\{\left[\frac{1}{\omega C}-\omega L\right]\right\}}{R^2+\left[\frac{1}{\omega C}-\omega L\right]^2}\cos(\omega t+\phi) + \frac{V[R]}{R^2+\left[\frac{1}{\omega C}-\omega L\right]^2}\sin(\omega t+\phi)} \quad \text{--- (16)}$$

→ eqn 18 ← eqn (17)

(16) (22)

Impedance triangle:-



$$Z = R + j\left[\frac{1}{\omega C} - \omega L\right]$$

$$\cos\varphi = \frac{R}{Z} \qquad \boxed{\therefore R = Z\cos\varphi} \text{ — (17)}$$

$$\sin\varphi = \frac{\frac{1}{\omega C} - \omega L}{Z}$$

$$\boxed{\therefore \frac{1}{\omega C} - \omega L = Z\sin\varphi} \text{ — (18)}$$

$$\boxed{\tan\varphi = \frac{\left[\frac{1}{\omega C} - \omega L\right]}{R}} \text{ — (19)}$$

$$\boxed{Z^2 = R^2 + \left[\frac{1}{\omega C} - \omega L\right]^2} \text{ — (20)}$$

sub (17) & (18), (20) in equation no. (16)

$$I_p = \frac{V \cancel{Z}}{Z^2}\sin\varphi\cos(\omega t + \phi) + \frac{V\cancel{Z}}{Z^2}\cos\varphi\sin(\omega t + \phi)]$$

$$I_p = \frac{V}{Z}\left[\sin\varphi\cos(\omega t + \phi) + \cos\varphi\sin[\omega t + \phi]\right]$$

[$\because \sin(A+B) = \sin A\cos B + \cos A\sin B$]

$$\boxed{I_p = \frac{V}{Z}\left[\sin(\omega t + \phi + \varphi)\right]} \text{ —— (21)}$$

sub (7) & (21) in equation (3)

$$(3) \Rightarrow \boxed{I = I_c + I_p}$$

$$\boxed{\therefore I = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t} + \frac{V}{Z}\left[\sin(\omega t + \phi + \varphi)\right]} \text{ —— (22)}$$

(22) $\Rightarrow$ $$\boxed{I = \underbrace{\left[c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t}\right]}_{\text{Transient Term}} + \underbrace{\left[\frac{V_m}{Z}\left[\sin(\omega t + \phi + \psi)\right]\right]}_{\text{Steady state term}}}$$

* Based on $\left[\frac{R}{2L}\right]^2$ & $\frac{1}{LC}$ Three different current equation may arise.

Case i:- If $\left[\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}\right]$

Then $$\lambda_1 = -\frac{R}{2L} + \sqrt{\left[\frac{R}{2L}\right]^2 - \frac{1}{LC}}$$

$$\lambda_2 = -\frac{R}{2L} - \sqrt{\left[\frac{R}{2L}\right]^2 - \frac{1}{LC}}$$

The value of λ_1 & λ_2 $\rightarrow$ Real & Negative

(Distinct roots)

$\therefore$ $$\boxed{\begin{aligned}\lambda_1 &= \alpha + \beta \\ \lambda_2 &= \alpha - \beta\end{aligned}}$$ where $\alpha = -\frac{R}{2L}$

$$\beta = \left[\frac{R}{2L}\right]^2 - \frac{1}{LC}$$

$\therefore$ Two different roots

$$\therefore I = c_1 e^{(\alpha+\beta)t} + c_2 e^{(\alpha-\beta)t} + \frac{V}{Z}\sin\left[\omega t + \phi + \psi\right]$$

$$\boxed{\therefore I = e^{\alpha t}\left[c_1 e^{\beta t} + c_2 e^{-\beta t}\right] + \frac{V}{Z}\sin(\omega t + \phi + \psi)}$$ $\rightarrow$ (23)

* Oscillation will be {over damped.} [Two exponential term]

Case ii If $\left[\frac{R}{2L}\right]^2 = \frac{1}{LC}$

$$\lambda_1 = \frac{-R}{2L} + \sqrt{\left[\frac{R}{2L}\right]^2 - \left[\frac{R}{2L}\right]^2} \quad \left[\because \left[\frac{R}{2L}\right]^2 = \frac{1}{LC}\right]$$

$$\lambda_2 = -\frac{R}{2L} - \sqrt{\left[\frac{R}{2L}\right]^2 - \left[\frac{R}{2L}\right]^2}$$

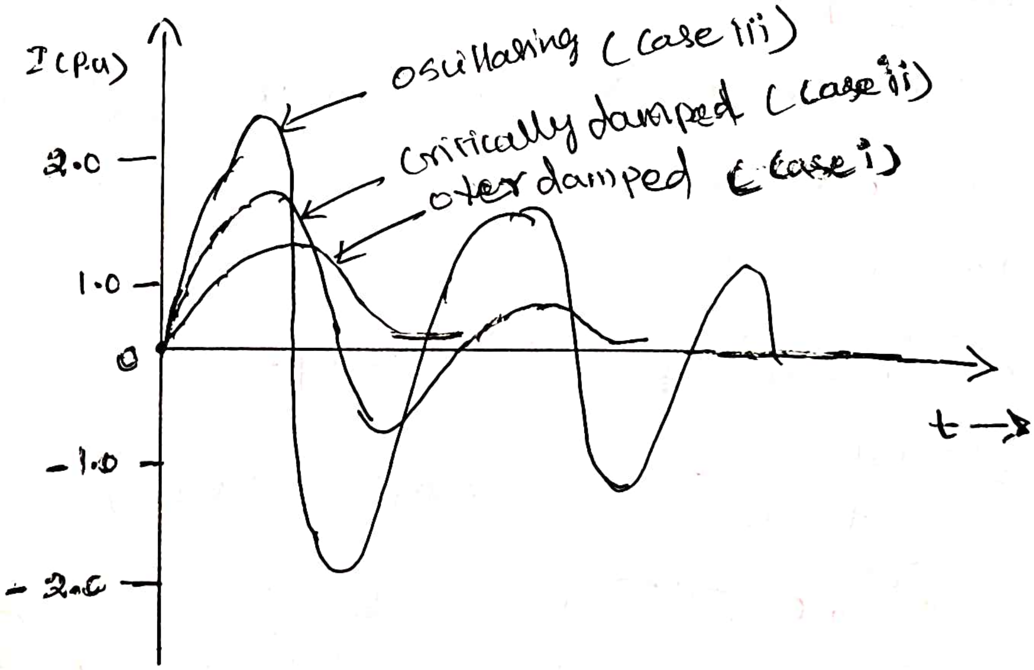
$$\boxed{\lambda_1 = \alpha + \beta} \quad \boxed{\lambda_2 = \alpha - \beta}$$

$$\boxed{\therefore \lambda_1 = \alpha \quad \lambda_2 = \alpha} \quad \boxed{\beta = 0} \quad \boxed{\alpha = -\frac{R}{2L}}$$

* Roots are real & equal
* Transient oscillation will be (critically damped).
* current equation becomes [$\because$ Two roots are equal]

$$I = c_1 e^{\alpha t} + c_2 e^{\alpha t} + \frac{V}{Z} \sin(\omega t + \phi + \psi)$$

$$\boxed{I = e^{\alpha t}[c_1 + c_2] + \frac{V}{Z} \sin(\omega t + \phi + \psi)} \longrightarrow (24)$$



Case iii If $\left[\frac{R}{2L}\right]^2 < \frac{1}{LC}$

W.K.T

$$\lambda_1 = -\frac{R}{2L} + \sqrt{\left[\frac{R}{2L}\right]^2 - \frac{1}{LC}}$$

$$\lambda_2 = -\frac{R}{2L} - \sqrt{\left[\frac{R}{2L}\right]^2 - \frac{1}{LC}}$$

$\frac{1}{LC} > \left[\frac{R}{2L}\right]^2$

$$\therefore \lambda_1 = -\frac{R}{2L} + \sqrt{-\left[\left(\frac{1}{LC} - \left[\frac{R}{2L}\right]^2\right)\right]} \qquad [i^2 = -1]$$

$$= -\frac{R}{2L} + \sqrt{i^2\left[\left[\frac{1}{LC}\right] - \left[\frac{R}{2L}\right]^2\right]}$$

$$\lambda_1 = -\frac{R}{2L} + i\sqrt{\frac{1}{LC} - \left[\frac{R}{2L}\right]^2}$$

Illly $\lambda_2 = -\frac{R}{2L} - i\sqrt{\frac{1}{LC} - \left[\frac{R}{2L}\right]^2}$

$$\boxed{\begin{aligned}\lambda_1 &= \alpha + j\beta \\ \lambda_2 &= \alpha - j\beta\end{aligned}}$$

where $\alpha = -\frac{R}{2L}$; $\beta = \sqrt{\frac{1}{LC} - \left[\frac{R}{2L}\right]^2}$

* The value of λ_1 & λ_2 are complex conjugate
* The transient oscillation will be oscillatory and the equation of current becomes

$$I = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t} + \frac{V}{Z}\left[\sin(\omega t + \phi + \psi)\right]$$

$$I = c_1 e^{(\alpha + j\beta)t} + c_2 e^{(\alpha - j\beta)t} + \frac{V}{Z}\sin\left[(\omega t + \phi + \psi)\right]$$

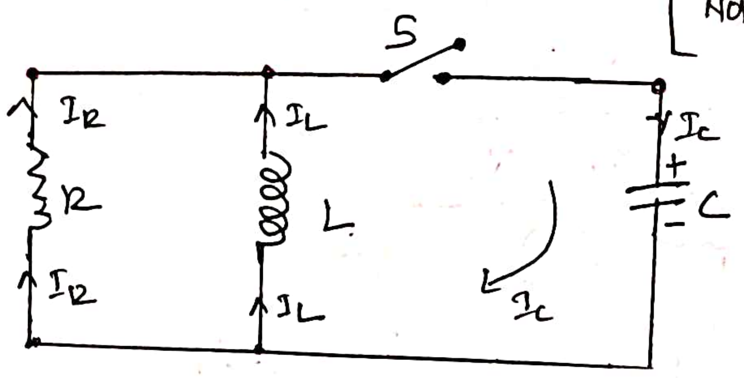
$$I = e^{\alpha t}\left[c_1 e^{j\beta t} + c_2 e^{-j\beta t}\right] + \frac{V}{Z}\sin\left[\omega t + \phi + \psi\right]$$

$$\boxed{I = e^{\alpha t}\left[c_1 \cos\beta t + c_2 \sin\beta t\right] + \frac{V}{Z}\sin\left[(\omega t + \phi + \psi)\right]}$$

Basic transforms of parallel RLC circuit:-

Consider the following parallel R-L-C circuit

[Note:- Source free]



* When a switch 'S' closed in the capacitor branch allowing the capacitor 'C' to discharge through 'R' & 'L' elements.

* Now the capacitor 'C' is charged with the voltage of 'Vc' & the current flow through the capacitor.

* Now the current in the capacitor is equal to the sum of the currents in the other elements.

Apply KCL to the circuit:-

KCL:-

Sum of the currents entering the node is equal to the sum of the current leaving the node

ie

$$\boxed{I_R + I_L + I_c = 0} \quad \text{--- (1)}$$

$$\boxed{-I_c = I_R + I_L} \quad \text{--- (2)}$$

$$\boxed{I_R = \frac{V_c}{R}} \;; \quad \text{(3)} \qquad \boxed{I_c = C \cdot \frac{d}{dt} V_c} \quad \text{--- (4)}$$

Sub (3) & (4) in equation number (2)

(2) $\Rightarrow$ $$\boxed{-C\frac{d}{dt}V_C = I_L + \frac{V_C}{R}} \quad \text{--- (5)}$$

In parallel circuit voltage same i.e.

$$\boxed{V_R = V_L = V_C}$$

Now Let $\boxed{V_L = V_C}$ --- (6)

$$\boxed{\therefore V_L = L\cdot\frac{dI_L}{dt}} \quad \text{--- (7)}$$

Sub (7) in (6)

i.e. $$\boxed{V_C = L\cdot\frac{d}{dt}I_L} \quad \text{--- (8)}$$

Sub (8) in equation no. (5)

$$-C\cdot\frac{d}{dt}\left[L\cdot\frac{d}{dt}I_L\right] = I_L + \frac{L}{R}\cdot\frac{d}{dt}I_L$$

$$-CL\frac{d^2}{dt^2}I_L = I_L + \frac{L}{R}\cdot\frac{d}{dt}I_L$$

$$-\frac{d^2}{dt^2}I_L = \frac{I_L}{CL} + \frac{\cancel{L}}{R}\times\frac{1}{C\cancel{L}}\cdot\frac{d}{dt}I_L$$

$$-\frac{d^2}{dt^2}I_L = \frac{I_L}{CL} + \frac{1}{RC}\cdot\frac{d}{dt}I_L$$

Re arranging $\rightarrow$ second term

$$\boxed{\frac{d^2}{dt^2}I_L + \frac{1}{RC}\cdot\frac{d}{dt}I_L + \frac{I_L}{CL} = 0} \quad \text{--- (9)}$$

W.K.T for series circuit

$$\boxed{\frac{d^2}{dt^2}I_L + \frac{R}{L}\frac{d}{dt}I_L + \frac{I_L}{CL} = 0} \quad \text{--- (10)}$$

* It is to be noted that the difference between equation (9) & (10) is in the co-efficient of their second term only.

* Parallel time constant $\boxed{T_p = RC}$ —— (11)

* Series time constant $\boxed{T_s = \frac{L}{R}}$ —— (12)

* The product of these two time constants gives the square of angular period of under damped circuit.

$$T_p T_s = LC = T^2$$

$$T_p T_s = \cancel{R} C \times \frac{L}{\cancel{R}}$$

$$\therefore T_p T_s = LC$$

$$\boxed{T^2 = LC} \quad \text{—— (13)}$$

$$\boxed{T = \sqrt{LC}} \quad \text{—— (14)}$$

* Substitute (11) & (13) in equation no:-9

$$\boxed{\frac{d^2}{dt^2} I_L + \frac{1}{T_p} \cdot \frac{d}{dt} I_L + \frac{I_L}{T^2} = 0} \quad \text{—— (15)}$$

[Here no voltage source $\therefore V_s = 0$]

* Taking laplace transform of equation number (15)

$$s^2 I_L(s) + \frac{1}{T_p} s I_L(s) + \frac{1}{T^2} I_L(s) - s I_L(0) - \frac{1}{T_p} I_L(0) - I_L'(0) = 0$$

$$\boxed{\left[s^2 + \frac{1}{T_p} s + \frac{1}{T^2}\right] I_L(s) = \left[s + \frac{1}{T_p}\right] I_L(0) + I_L'(0)}$$

$$\left[\because \left(\frac{d}{dt}\right) = s\right]$$

* But the initial current $\boxed{I_L(0) = 0}$ —— (17)

From equation no. 8 (8) $\Rightarrow$ $\boxed{V_C = L \cdot \dfrac{dI_L}{dt}}$ —— (18)

$$\boxed{\frac{dI_L}{dt} = \frac{V_C}{L}} \quad \text{—— (19)}$$

$$\left[\because \frac{d}{dt} I_L = I_L' \right.$$

Initial cond. $I_L'(0)$]

$$\boxed{I_L'(0) = \frac{V_C(0)}{L}} \quad \text{—— (20)}$$

$$\left. \therefore I_L'(0) = \frac{V_C(0)}{L} \right]$$

Substitute (17) & (20) in equation number (16)

$$(16) \Rightarrow \left[s^2 + \frac{1}{T_p} s + \frac{1}{T^2} \right] I_L(s) = \frac{V_C(0)}{L}$$

$$\boxed{\therefore I_L(s) = \frac{V_C(0)}{L} \left[\frac{1}{s^2 + \frac{s}{T_p} + \frac{1}{T^2}} \right]} \quad \text{—— (21)}$$

* To find out the inverse Laplace transform of equation no. (21) ~~the~~ the following steps are followed

The solution of equation no:- 21

Assume s_1 & s_2 are the roots of equation

$$\boxed{s^2 + \frac{1}{T_p} s + \frac{1}{T^2} = 0} \quad \text{—— (22)}$$

$$\left[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \right.$$

$$\therefore s = \frac{-\frac{1}{T_p} \pm \sqrt{\left(\frac{1}{T_p}\right)^2 - 4 \times 1 \times \frac{1}{T^2}}}{2 \times 1}$$

Here $a = 1$, $b = \frac{1}{T_p}$

$$\left. c = \frac{1}{T^2} \right]$$

$$\boxed{\therefore s = -\frac{1}{2T_p} \pm \frac{1}{2} \sqrt{\frac{1}{T_p^2} - \frac{4}{T^2}}} \quad \text{—— (23)}$$

20

$$\left.\begin{aligned} s_1 &= \frac{1}{2T_p} + \frac{1}{2}\sqrt{\frac{1}{T_p^2} - \frac{4}{T^2}} \\ s_2 &= \frac{1}{2T_p} - \frac{1}{2}\sqrt{\frac{1}{T_p^2} - \frac{4}{T^2}} \end{aligned}\right\} \text{— (24)}$$

solution of $\dfrac{1}{s^2 + \frac{1}{T_p}s + \frac{1}{T^2}} = \dfrac{A}{s-s_1} + \dfrac{B}{s-s_2}$

$$\Rightarrow \boxed{\frac{1}{(s-s_1)(s-s_2)} = \frac{A}{s-s_1} + \frac{B}{s-s_2}} \text{ — (25)}$$

$$\frac{1}{\cancel{(s-s_1)}\cancel{(s-s_2)}} = \frac{A(s-s_2) + B(s-s_1)}{\cancel{(s-s_1)}\cancel{(s-s_2)}}$$

$$\boxed{A(s-s_2) + B(s-s_1) = 1} \text{ — (26)}$$

put $\boxed{s = s_1}$ in equation no:- 26

$$\therefore A(s_1 - s_2) + B(s_1 - s_1) = 1$$

$$A(s_1 - s_2) + 0 = 1$$

$$\boxed{\therefore A = \frac{1}{s_1 - s_2}} \text{ — (27)}$$

put $s = s_2$ in equation no:- 26

$$A(s_2 - s_2) + B(s_2 - s_1) = 1$$

$$0 + B(s_2 - s_1) = 1$$

$$\boxed{B = \frac{1}{(s_2 - s_1)}} \text{ — (28)}$$

sub (27) & (28) in equation number (25)

$$\frac{1}{(s-s_1)(s-s_2)} = \frac{1}{(s-s_1)(s_1-s_2)} + \frac{1}{(s-s_2)(s_2-s_1)}$$

$$= \frac{1}{(s-s_1)(s_1-s_2)} - \frac{1}{(s_1-s_2)(s-s_2)}$$

$$= \frac{1}{(s_1-s_2)}\left[\frac{1}{s-s_1} - \frac{1}{s-s_2}\right] \quad \text{— (29)}$$

from 24 we get

$$s_1 - s_2 = \cancel{\frac{1}{2T_p}} + \frac{1}{2}\sqrt{\frac{1}{T_p^2}-\frac{4}{T^2}} - \cancel{\frac{1}{2T_p}} + \frac{1}{2}\sqrt{\frac{1}{T_p^2}-\frac{4}{T^2}}$$

$$\boxed{s_1 - s_2 = \sqrt{\frac{1}{T_p^2}-\frac{4}{T^2}}} \quad \text{— (30)}$$

sub(30) in (29)

$$\boxed{\frac{1}{s^2+\frac{s}{T_p}+\frac{1}{T^2}} = \frac{1}{\sqrt{\frac{1}{T_p^2}-\frac{4}{T^2}}}\left[\frac{1}{s-s_1} - \frac{1}{s-s_2}\right]} \quad \text{— (31)}$$

put equation no. 31 in equation number (21)

$$\boxed{I_L(s) = \frac{V_c(0)}{L\sqrt{\frac{1}{T_p^2}-\frac{4}{T^2}}}\left[\frac{1}{s-s_1} - \frac{1}{s-s_2}\right]} \quad \text{— (32)}$$

Taking inverse Laplace transform of equation no. 32

$$\boxed{I_L(t) = \frac{V_C(0)}{L\left[\frac{1}{T_p^2} - \frac{4}{T^2}\right]^{1/2}} \cdot \left(e^{s_1 t} - e^{s_2 t}\right)} \quad \text{—— (33)}$$

Condition :-

(i) If $\frac{1}{T_p^2} > \frac{4}{T^2}$ the roots 's_1 & s_2' are real

(ii) If $\frac{1}{T_p^2} < \frac{4}{T^2}$ the root 's_1 & s_2' are complex — 34

→ These conditions ~~are~~ can be expressed in terms of the parameters 'η'

$$\boxed{\eta = \frac{R}{Z_n}}$$ where $Z_n \rightarrow$ Surge Impedance

(or)

$$\boxed{\eta = \frac{T_p}{T}} \quad 35$$

Condition 1 :- can be written as (in terms of η)

If $\boxed{\frac{1}{T_p^2} > \frac{4}{T^2}}$ then $\boxed{\eta < \frac{1}{2} = \text{real roots}}$ - 36

ie $\frac{1}{4} > \frac{T_p^2}{T^2}$ ie $\frac{T_p^2}{T^2} < \frac{1}{4}$

ie $\frac{T_p}{T} < \sqrt{\frac{1}{4}}$ $\left[\because \eta = \frac{T_p}{T}\right]$

$\frac{T_p}{T} < \frac{1}{2}$

ie $\boxed{\eta < \frac{1}{2}}$

Condition 2 :- can be written in terms of η as

If $\frac{1}{T_p^2} < \frac{4}{T^2}$ then $\eta > \frac{1}{2}$ then root are complex

i.e $\frac{1}{4} < \frac{T_p^2}{T^2}$

$$\frac{T_p^2}{T^2} > \frac{1}{4}$$

$$\frac{T_p}{T} > \sqrt{\frac{1}{4}} \qquad \left[\because \eta = \frac{T_p}{T}\right]$$

$\boxed{\therefore \eta > \frac{1}{2}}$ roots are complex

So that for the condition where $\boxed{\eta > \frac{1}{2}}$ (under damped)

Now the roots s_1 & s_2 can be written in terms of $\boxed{\eta}$

$$s_1 = -\frac{1}{2T_p} + \frac{1}{2}\sqrt{\frac{1}{T_p^2} - \frac{4}{T^2}}$$

i.e $$\boxed{s_1 = -\frac{1}{2T_p}\left[1 - j(4\eta^2 - 1)^{1/2}\right]} \quad \text{(37)}$$

$$\boxed{s_2 = -\frac{1}{2T_p} - \frac{1}{2}\sqrt{\frac{1}{T_p^2} - \frac{4}{T^2}}}$$

i.e $$\boxed{s_2 = -\frac{1}{2T_p}\left[1 + j(4\eta^2 - 1)^{1/2}\right]} \quad \text{(38)}$$

Now The equation (33) becomes

$$\boxed{I_L(t) = \frac{V_c(0)}{L} \frac{2T_p e^{-t/2T_p}}{(4\eta^2-1)^{1/2}} \sin (4\eta^2-1)^{1/2} \cdot \frac{t}{2T_p}} \quad \text{—(39)}$$

when the roots are real $\eta < \frac{1}{2}$ (over damped)

$$\left\{ \begin{aligned} S_1 &= -\frac{1}{2T_p}\left[1-(1-4\eta^2)^{1/2}\right] \\ S_2 &= -\frac{1}{2T_p}\left[1+(1-4\eta^2)^{1/2}\right] \end{aligned} \right\} \quad \text{—(40)}$$

Now

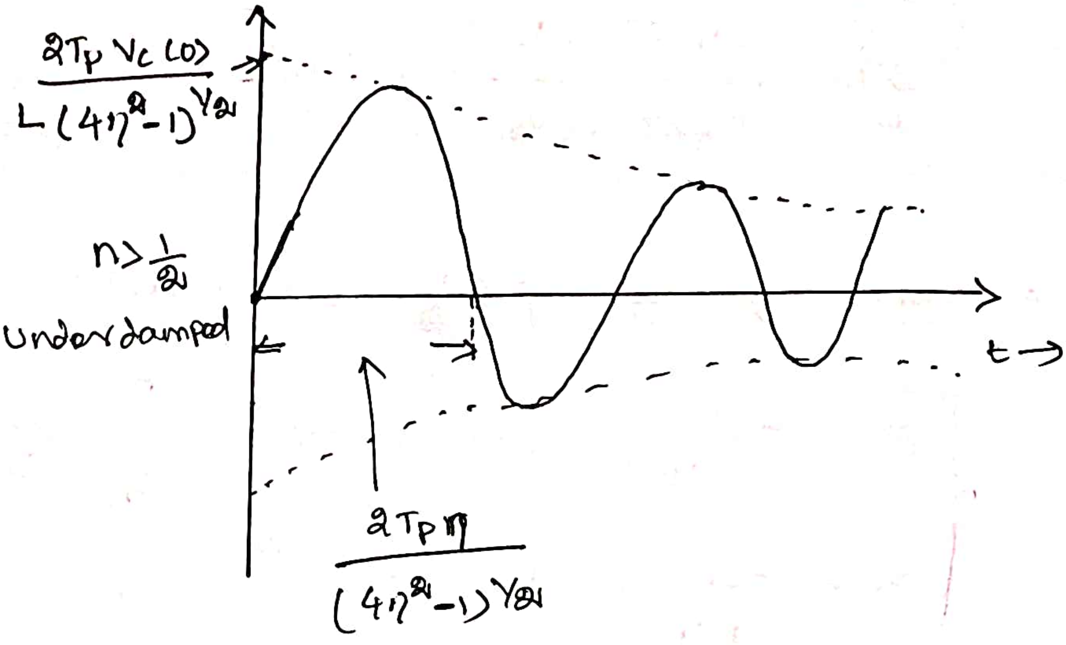
$$\boxed{I_L(t) = \frac{V_c(0)}{L} \frac{2T_p e^{-t/2T_p}}{(1-4\eta^2)^{1/2}} \sinh (1-4\eta^2)^{1/2} \cdot \frac{t}{2T_p}} \quad \text{—(41)}$$

when $\eta = \frac{1}{2}$ (critically damped)

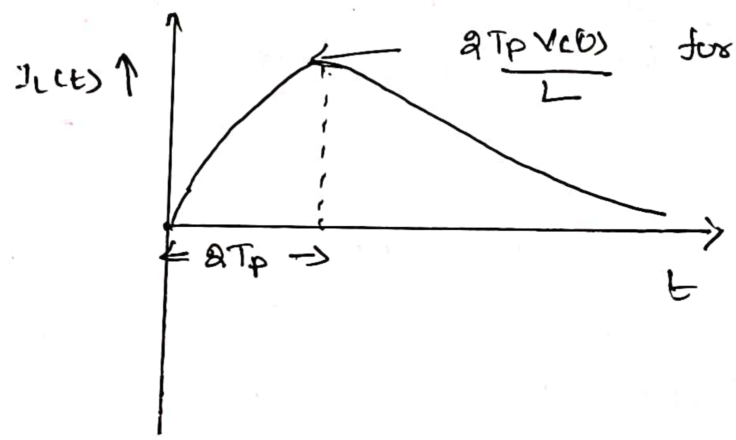
$$\boxed{I_L(t) = \frac{V_c(0)}{L} t e^{-\frac{t}{2T_p}}} \quad \text{—(42)}$$

[Note:- The detailed expansion of equations from (37 to 42) attached as a Annexure page Refer that for best understanding]

Wave forms:- (Under damped $\eta > \frac{1}{2}$)

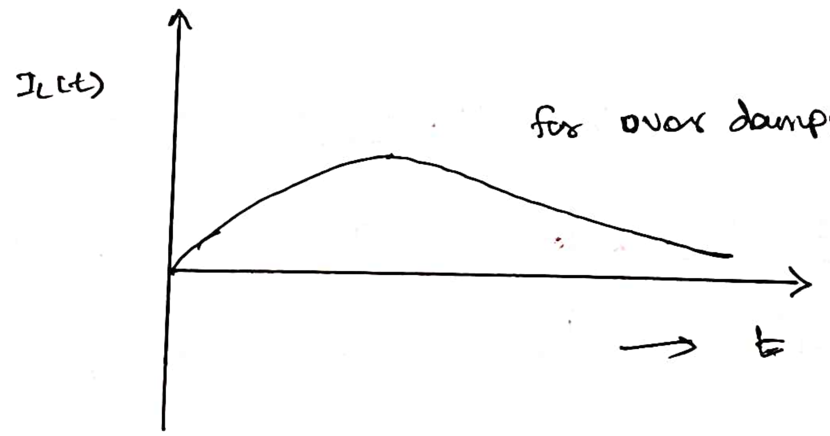


$\eta = \frac{1}{2}$ Critically damped:-



for critically damping $\boxed{\eta = \frac{1}{2}}$

$\eta < \frac{1}{2}$ Over damped:-



for over damping $\eta < \frac{1}{2}$

ANNEXURE

BASIC TRANSFORM OF PARALLEL RLC CIRCUIT:-

Expansion of Equation No. 37 to 42:-

$\eta > \frac{1}{2}$ i.e $\frac{1}{T_p^2} < \frac{4}{T^2}$

$$\therefore \quad S_1 = -\frac{1}{2T_p} + \frac{1}{2}\sqrt{\frac{1}{T_p^2} - \frac{4}{T^2}}$$

$$= -\frac{1}{2T_p} + \frac{1}{2}\sqrt{-\left[\frac{4}{T^2} - \frac{1}{T_p^2}\right]}$$

$$= -\frac{1}{2T_p} + \frac{1}{2}\sqrt{\frac{j^2}{T_p^2}\left[\frac{4}{T^2} \times T_p^2 - 1\right]}$$

$$= -\frac{1}{2T_p} + \frac{1}{2T_p} j\sqrt{\left[4 \times \left[\frac{T_p}{T}\right]^2 - 1\right]}$$

$$= -\frac{1}{2T_p} + \frac{1}{2T_p} j\sqrt{4\eta^2 - 1} \qquad \left[\because \eta = \frac{T_p}{T}\right]$$

$$S_1 = -\frac{1}{2T_p} + \frac{1}{2T_p} j\sqrt{(4\eta^2 - 1)}$$

$$\boxed{S_1 = -\frac{1}{2T_p}\left[1 - j(4\eta^2 - 1)^{1/2}\right]} \quad \text{—— (37)}$$

then $|||$ly

$$\boxed{S_2 = -\frac{1}{2T_p}\left[1 + j(4\eta^2 - 1)^{1/2}\right]} \quad \text{—— (38)}$$

Sub (37 a 38) in equation (33)

$$I_L(t) = \frac{V_c(0)}{L\left[\frac{1}{T_p^2} - \frac{4}{T^2}\right]^{1/2}} \cdot \left[e^{s_1 t} - e^{s_2 t}\right]$$

$$= \frac{V_c(0)}{L\left[\frac{1}{T_p^2} - \frac{4}{T^2}\right]^{1/2}} \left[e^{\left[1-j(4\eta^2-1)^{1/2}\right]\left[\frac{-t}{2T_p}\right]} - e^{-\frac{t}{2T_p}\left[1+j(4\eta^2-1)^{1/2}\right]} \right]$$

$$= \frac{V_c(0)}{L\left[\frac{1}{T_p^2} - \frac{4}{T^2}\right]^{1/2}} \left[e^{-\frac{t}{2T_p}} \cdot e^{\frac{j(4\eta^2-1)^{1/2} t}{2T_p}} - e^{-\frac{t}{2T_p}} \cdot e^{\frac{-j(4\eta^2-1)^{1/2} t}{2T_p}} \right]$$

$$= \frac{\cancel{V_c(0)} \cdot e^{-\frac{t}{2T_p}}}{L\left[\frac{j^2}{T_p^2}\left[\frac{4}{T^2} \cancel{\times 1}\right.\right.}$$

$$= \frac{V_c(0)\ e^{-\frac{t}{2T_p}}}{L\left[\frac{j^2}{T_p^2}\left[\frac{4 \times T_p^2}{T^2} - 1\right]\right]^{1/2}} \left[e^{\frac{j(4\eta^2-1)t}{2T_p}} - e^{\frac{-j(4\eta^2-1)^{1/2} t}{2T_p}} \right]$$

$$= \frac{V_c(0) \cdot T_p \cdot e^{-t/2T_p}}{Lj\left[4\eta^2-1\right]^{1/2}} \left[\cos\left[\frac{4\eta^2-1)\cancel{t}}{2T_{\cancel{p}}}\right] + j\sin\left[\frac{(4\eta^2-1)t}{2T_p}\right] - \cos\left[\frac{(4\eta^2-\cancel{1})t}{2T_{\cancel{p}}}\right] + j\sin\frac{(4\eta^2-1)t}{2T_p} \right]$$

$$= \frac{V_c(0) \cdot T_p \cdot e^{-t/2T_p}}{Lj\left[4\eta^2-1\right]^{1/2}} \left[2j \sin(4\eta^2-1)\frac{t}{2T_p} \right]$$

$$\boxed{I_L(t) = \frac{V_c(0)\; 2T_p\; e^{-t/2T_p}}{L\,[4\eta^2-1]^{1/2}} \sin(4\eta^2-1)\cdot\frac{t}{2T_p}} \quad \text{— (39)}$$

$$\left[\because e^{j\theta} = \cos\theta + j\sin\theta \right.$$
$$e^{-j\theta} = \cos\theta - j\sin\theta$$
$$\left. e^{j\theta} - e^{-j\theta} = 2j\sin\theta \right]$$

When $\eta < \frac{1}{2}$ ie $\frac{1}{T_p^2} > \frac{4}{T^2}$:-

Then

$$S_1 = -\frac{1}{2T_p} + \frac{1}{2}\sqrt{\frac{1}{T_p^2} - \frac{4}{T^2}}$$

$$= -\frac{1}{2T_p} + \frac{1}{2}\left[\frac{1}{T_p^2}\left[1 - \frac{4}{T^2}\times T_p^2\right]\right]^{1/2}$$

$$= -\frac{1}{2T_p} + \frac{1}{2T_p}\left[[1-4\eta^2]^{1/2}\right] \quad \left[\because \eta = \frac{T_p}{T}\right]$$

$$S_1 = -\frac{1}{2T_p} + \frac{1}{2T_p}\left[[1-4\eta^2]^{1/2}\right]$$

$$S_1 = \frac{-1}{2T_p}\left[1 - [1-4\eta^2]^{1/2}\right]$$

lly then — (40)

$$S_2 = -\frac{1}{2T_p}\left[1 + (1-4\eta^2)^{1/2}\right]$$

25 (e)

$$\boxed{I_L(t) = \frac{V_C(0)}{L}\,\frac{e^{-\frac{t}{2T_P}}\,2T_P}{[1-4\eta^2]^{1/2}}\left[\sinh(1-4\eta^2)^{1/2}\cdot\frac{t}{2T_P}\right]}$$

— (41)

$$\left[\because \sinh = \frac{e^{x}-e^{-x}}{2}\right]$$

when $\eta = \frac{1}{2}$ ie $\frac{1}{T_P^2} = \frac{4}{T^2}$:-

$$S_1 = -\frac{1}{2T_P} + \frac{1}{2}\sqrt{\frac{1}{T_P^2} - \frac{4}{T^2}}$$

$$S_1 = -\frac{1}{2T_P} + 0 \qquad \left[\because \frac{1}{T_P^2} = \frac{4}{T^2}\right]$$

$$\boxed{\therefore S_1 = S_2 = -\frac{1}{2T_P}}$$

$$\therefore I_L(t) = \frac{V_C(0)}{L}\left[e^{S_1 t} - e^{S_2 t}\right]$$

$$\boxed{I_L(t) = \frac{V_C(0)}{L}\,t\,e^{-t/2T_P}} \quad \text{—— (42)}$$

Effects of Transients on power system

1.6 DIFFERENT TYPES OF POWER SYSTEM TRANSIENTS

A) Based on their origin:-

There are two types namely

(i) Atmospheric origin - (Lightning)

(ii) Switching origin - (Switching operations)

B) Based on the mode of Generation of Transient:-

(i) Electro magnetic:-

* Changes in voltages and currents are usually caused by closing or opening of circuit Breakers, power Electronic equipment's, Equipment failure (or) faults Lightning strokes etc.,

(ii) Electro mechanical:-

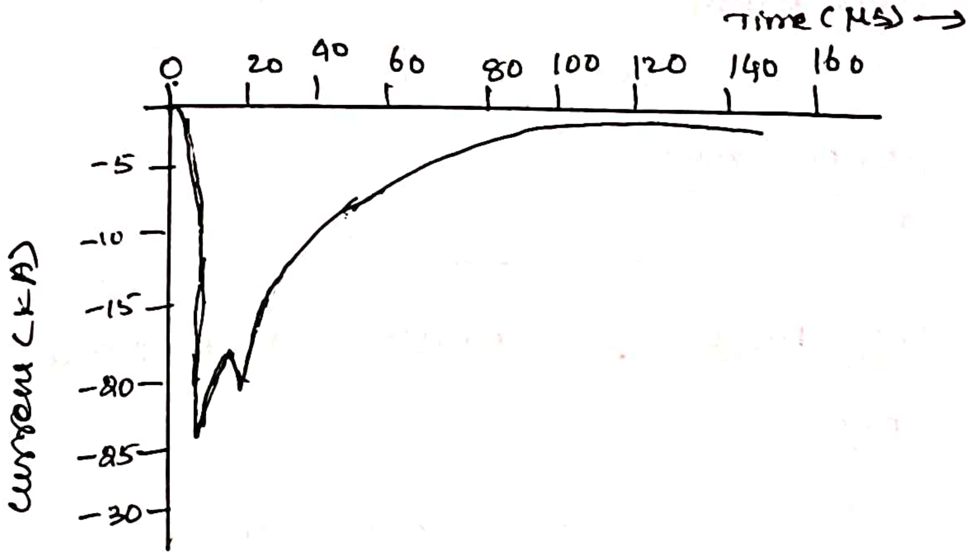
* Transients are caused by mismatch between power Production and consumption causing the generator to either speed up (or) slow down compared to its normal rotational speed.

(C) Transients Based on the nature of waveform:-

(i) Impulsive:-

* An Impulsive transient is a sudden non-power frequency event in which the steady-state condition of voltage, current (or) both change suddenly.

* The most common sources of Impulsive transients are {lightning} and Electrostatic discharge



* The change is unidirectional in polarity i.e either positive (or) negative
* Impulsive transients are characterized by rise and fall time
* From the fig. the maximum amplitude of the transient current is $\boxed{23\,kA}$ & duration of transient current is less than $\boxed{10\,\mu s}$

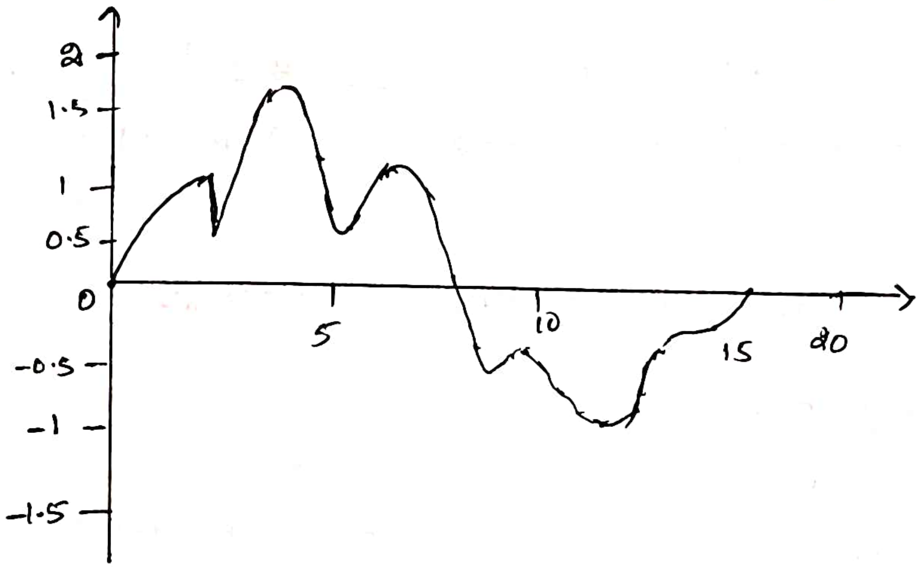
Types of Impulsive Transients:-

Impulsive Transient	Rise time	Duration
Nanosecond	5ns	Less than 50ns
Microsecond	1µs	50ns to 1ms
milli second	0.1ms	more than 1ms

(ii) Osillatory Transients:-

* An osillatory transient is a sudden non-power frequency event in which the steady state condition of voltage, current or both changes polarity rapidly.
* change in polarity is bidirectional.
* occurs due to resonances during switching
* frequency ranging from few KHZ to several MHZ

EX:- Switching shunt capacitors



Types of osillatory transient:-

(i) Low frequency (< 5KHZ)
(ii) Medium frequency (5-500KHZ)
(iii) High frequency (0.5-5MHZ)

S.I NO	Osillatory Transients	Spectral content	Typical duration	Typical voltage magnitude
(1)	Low frequency	< 5KHZ	0.3 - 50 ms	0-4 p.u
(2)	Medium fq	5-500KHZ	20 μs	0-8 pu
(3)	High fq	0.5-5MHZ	5 μs	0-4 pu

D) Depends upon the speed of transients:-

(i) Ultra fast Transients:-

* These types of lightning transients are caused by lightning and when inductive loading is cut-off

(ii) Medium fast Transients:-

* It's caused by abrupt short circuits in the system.

(iii) Slow Transient:-

* These transients are electromechanical in nature causing mechanical oscillations of rotors of synchronous machines

E) Depends on control of on the transients:-

(i) Single transient:-

* In this type of transient we are in position to open or close the switch at our discretion, therefore able to anticipate the consequences [Eg:- switching on fans, Etc]

(ii) Recurrent transients:-

* Occurs regularly as a commutation transient in the converting Equipment. [Ex:- Rectifiers, Invertors, etc]

(iii) Random Transient

* These transients are generated by extraneous operations beyond our control which appear in unpredictable manner. [Ex:- switching on parallel lines which is near to live conductor]

1.9 EFFECTS OF TRANSIENTS IN POWER SYSTEM:- 18 2/8

* Transients or surges are over voltage conditions which can result in damage to electrical equipment.

* The effect and severity depend on magnitude, duration and frequency.

* Low Energy transients can cause electrical Equipment to malfunction while high energy transients can cause damage to equipment.

Problems associated with transients can cause

a) Damage to Electronic Equipment

b) Unusual equipment damage due to insulation failures or flash-over

c) Total failure, lock-up (or) misoperation of computers (or) micro-processor based equipment.

d) This also has a harsh effect on IC (Integrated Circuits) and can result in getting them burnt out.

e) motors can be easily ~~heate~~ heated up which leads to Insulation failure

f) Hysteresis Loss is increased and lead to more current being injected into the motor for the same output.

g) Transient Voltage suppression equipment can double or triple the life of electrical and electronic equipment.

Electronic Equipment:-

* Equipment will malfunction and produces corrupted result.

* Improper specification and installation of Transient Voltage Surge suppression (TVSS) can aggravate the failures

* Efficiency of electronic devices will be reduced.

Motors:-

* Transient will make motors run at higher temperature.

* Result in micro-jogging leading to motor vibration excessive heat and noise

* Degrades the insulation of motors winding resulting to equipment failure

* Increases the motor's losses (hysteresis) and its operating temperature

Lights:-

* Fluorescent bulb and ballast failure

* Appearance of black rings at the fluorescent tube ends (Indicator of transients)

* Premature filament damage leading to failure of incandescent light.

Electrical Distribution Equipment:-

* Transients degrades the contacting surfaces of circuit breakers and switches
* Nuisance tripping of breakers due to false activation to a non-existent current demand
* Reduces transformer efficiency because of increased hysteresis losses produced by transients and can run hotter than normal.

Effects on semiconductors:-

* Frequently damage occurs when a high reverse voltage is applied to a non-conducting PN Junction.
* Excess leakage current can occur across the junction between the terminations on the die surface.
* This leads to damage of IC's

Other Effects on Electro mechanical Equipment:-

* The high voltage generated by a breaking current to an inductor with a mechanical switch will lead to a break down of the contacts.

Intermittent Interruptions:-

* When transients are inserted into a data or control network this leads to lost or corrupted data.
* These results in the load tripping off or its operates improperly.

Other Effects of Transients:-

* Chronic degradation
* Latent failures
* Catastrophic failures.

etc.

1.10 ROLL OF STUDY OF TRANSIENTS IN SYSTEM PLANNINH.

(30)

Power system planning is a process in which the aim is to decide the on now as well as upgrading existing system elements in a system to adequately satisfy the loads for a fore seen future and to work properly in any adverse conditions. The possible elements in a system are.

1. Generation facilities
2. substations
3. Transmission lines and /or cables
4. Capacitors
5. Reactors.

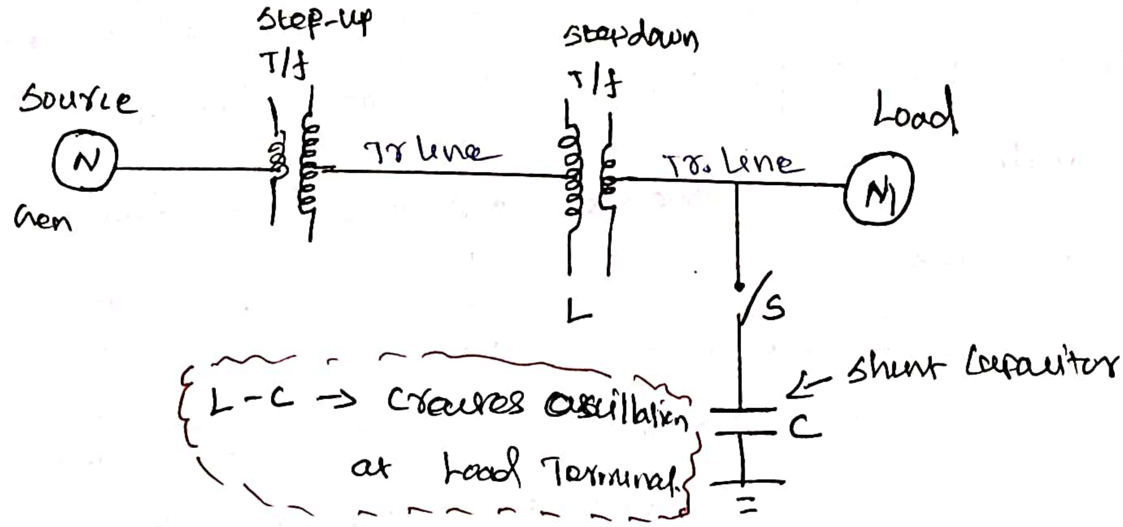
* The overvoltages generated due to lightning strikes and switching operations have the potential to damage the above said Electrical elements and result in large financial losses due to damaged equipment and lost production. These can be ~~ex~~ explained with the help of following Examples.

Example 1:- Shunt capacitor switching:.

* Shunt Capacitor switching is the main source of transients in Power system.

* These transient voltages can be magnified at the low voltage bus due to L-C combination termed

by step down transformer and low voltage capacitors connected to it causing significant interruption and damages on power systems.



Parameters that impact on transients due to capacitor switching

* Capacitor size (shunt)
* Step down transformer size
* Customer load characteristics

* Analysis of these parameters by system planners will provides both utility suppliers and customers an idea how to solve transient problems caused by shunt capacitor switching.

Example 2:- Energizing of long Transmission Line:-

* For a long distance over head transmission line the most serious problems are voltage surges in the power system which are the consequences of lightning and it produces over voltages which increases the risk of insulation failure

Remarks:-

* Accurate modelling of transmission line can help to lower the risk of failure caused by overvoltages.
* Sequential pole closures of circuit breaker (C.B) reduces the maximum over voltages at the receiving end of the line

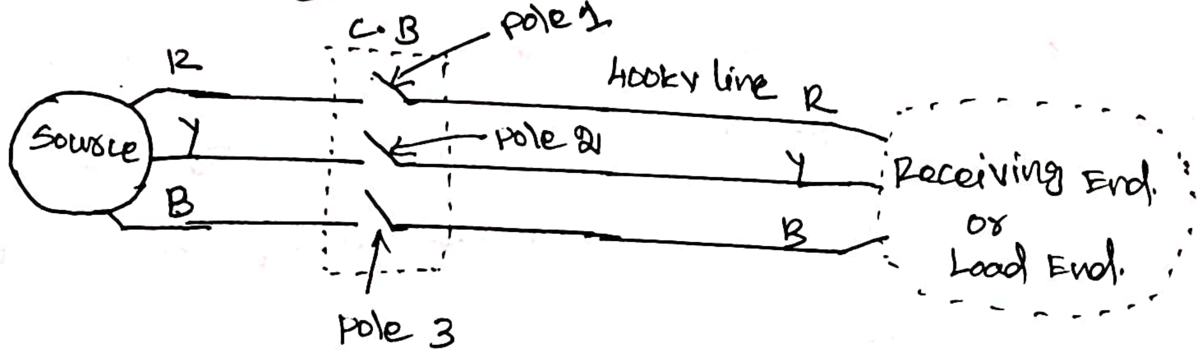


Fig:- pole clausures of Tr. line

* Pole-1 of C.B closes at time 't_1'
* Pole-2 of C.B Closes at time 't_2'
* Pole-3 of C.B Closes at time 't_3'
* $t_1 < t_2 < t_3$

* Sequential pole closure means switching on the poles of C.B one by one instead of simultaneous closure of all poles

Example 3:- Switching of solid state power loads:-

* Switching on the solid state power loads (Power Electronics loads) such as FACTS, SVCs, DC ties and rectifier loads causes steady state harmonic distortion and non-stationary harmonic distortion.

Rectifier loads:-

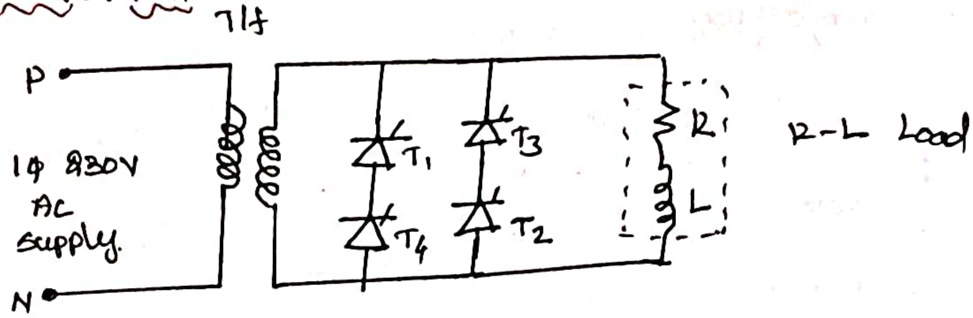


Fig:- Rectifier load.

* The efficient storge of waveforms which is caused due to switching 'on' Rectifier loads are used by the system planners to take a new look at the overall design of Transmission networks.
* The harmonics generated can be mitigated with the help of filters.

Conclusion:-

* The Proper care must be taken before installation of any Electrical Equipments in a power system.
* It should withstand the transients which is caused by different resources.
* Otherwise it will leads to the loss of cost, toll and affect the reliability of power system.
* ~~So~~ The study of transients in system planning is very important so that we can avoid the unwanted ~~error~~ events.

UNIT-II

SWITCHING TRANSIENTS

1.1 Switching Transients:-

Definition:-

Switching transients are initiated whenever a sudden change occurs in a power circuit especially during switching operations.

ie "closing and opening of switches"

* Switching operations can be classified into two categories
    (i) Energization
    (ii) De-Energization

(i) Energization:-

It includes the .

(a) Energization of lines
(b) Energization of cables
(c) Transformer Energization
(d) Energization of reactors and capacitor banks

(ii) De-Energization:-

It includes current interruption under faulted or unfaulted conditions. Ex:

(a) Switching off small capacitive currents
(b) Switching off of small inductive currents
(c) Fault initiation and clearing (d) Load rejection etc.

1.2 Switching over voltages in power system:-

→ Switching over voltages is one of the internal over voltages which may change the operating conditions of the power system.

1.2.1- Different causes of switching over voltages:-

→ When no load transmission line is suddenly switched on, the voltage on the line becomes twice of normal system voltage.

→ This voltage is transient in nature

→ When loaded line is switched off or interrupted voltage across the line becomes high causes overvoltage in the system.

* During insulation failure live conductor is suddenly earthed. This may also caused sudden overvoltage in the system.

* If emf wave produced by alternator is distorted, the trouble of resonance may occur due to 5'th or higher harmonics

* At this stage $\boxed{X_L = X_C}$, so the system reactance cancel each other, the system becomes purely resistive.

* This phenomenon is called resonance and at resonance the system voltage may be increased enough.

The study of switching transients involves to

(a) Determine the voltage stress on Equipment

(b) To select arrestor characteristics

(c) Calculate the transient recovery voltage across circuit breakers and

(d) Analyse the effectiveness of migitating devices [Eg:- Pre-insertion resistors or inductors)

1.3 RESISTANCE SWITCHING:-

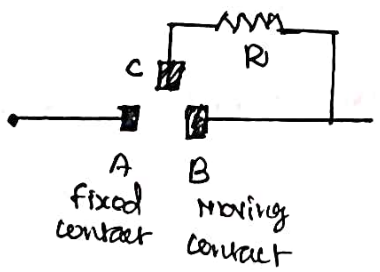
* "The connection of Resistance in parallel with the contact space or arc is called resistance switching."

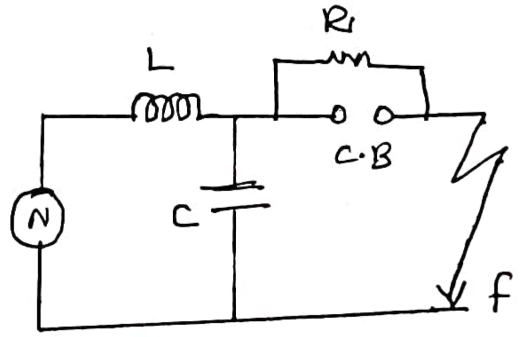
Definition:-

* Resistance switching in circuit Breaker refers to a method adopted for damping the over voltage transients due to current chopping, capacitive current breaking etc.,
* This Excessive voltage surges during circuit interruption can be prevented by the use of shunt resistance 'R' across the circuit breaker contacts
* This process is known as "resistance switching".

Circuit Diagram:-

Typical resistor connection





(b) Resistance switching Ckt.

Working:-

* when the fault occurs the contacts of the circuit breaker are open and an arc is struck between the contacts

* with the arc shunted by the resistance 'R', a part of arc current is diverted through the resistance.

* This results in the decrease of arc current and an increase in the rate of deionization of the arc path and resistance of arc.

* This will increase the current through shunt resistance. This process continues ~~un~~ untill the current through the arc is diverted through the resistance either completely or in major part.

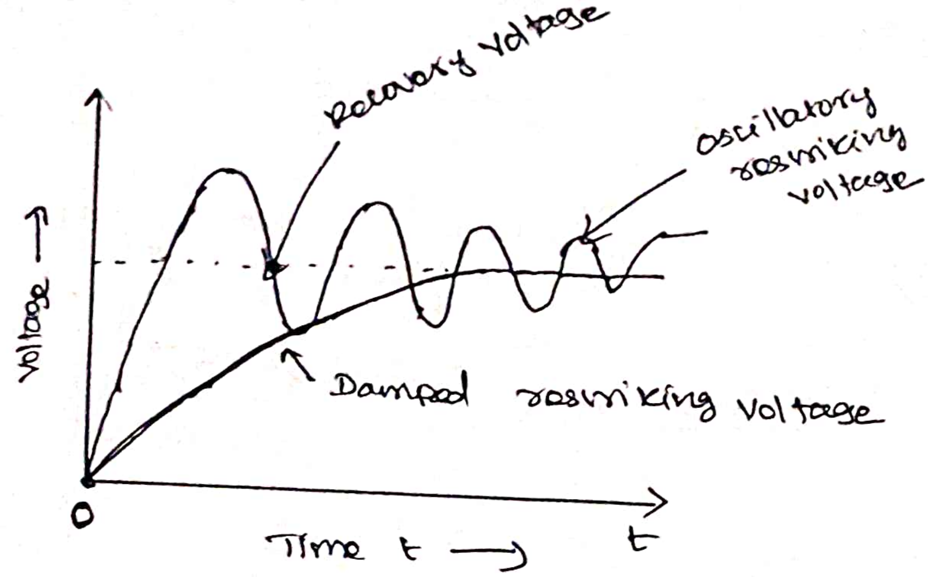
* If the small value of current remains in the arc then the path becomes so unstable that it is easily extinguished

* The resistance may be automatically switched in and arc current can be transferred. The time required for this process is less.

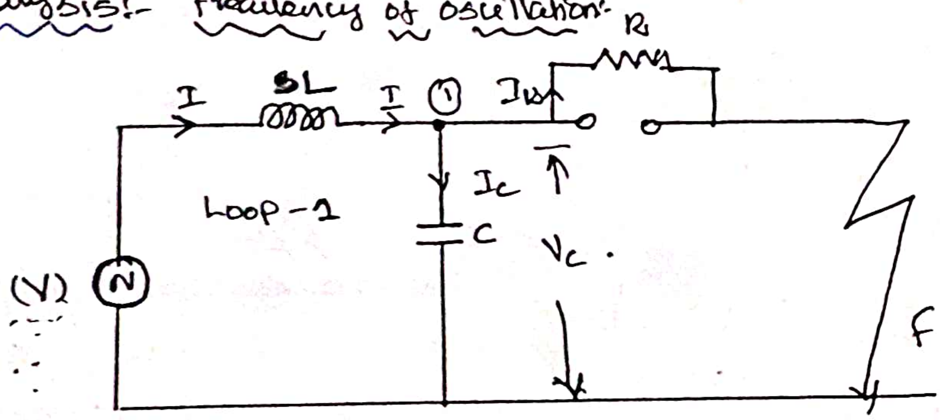
* The arc first appears across 'A' and 'B' point of circuit breaker contacts. Then the arc is transferred across the A and C

* The shunt resistance also ensures the effective damping of high frequency restriking transients.

Wave form:-



Analysis:- Frequency of oscillation:-



Consider the L-C circuit and its equivalent circuit is shown in fig.

Apply KVL in loop-1 : $\boxed{V = V_L + V_C}$

$$\boxed{L \cdot \frac{dI}{dt} + \frac{1}{C} \int I_c \cdot dt = V} \quad \text{------ (1)}$$

Apply KCL to the node between 'C' & 'R'

ie sum of current Entering node = Leaving the node

$$\boxed{I = I_R + I_C} \quad \text{------ (2)}$$

Sub (2) in (1)

$$L\cdot\frac{d}{dt}[I_c+I_R] + \frac{1}{c}\int I_c\cdot dt = V$$

$$\boxed{L\cdot\frac{dI_c}{dt} + L\cdot\frac{dI_R}{dt} + \frac{1}{c}\int I_c\cdot dt = V} \quad \text{.....(A)}$$

W.K.T $\quad I_c = c\cdot\frac{dV_c}{dt}$ .... (3)

diff.(3) w.r.to 't'

$$\boxed{\frac{dI_c}{dt} = c\cdot\frac{d^2V_c}{dt^2}} \quad \text{.........(4)}$$

$$\frac{dI_R}{dt} = \frac{d\left[\frac{V_c}{R}\right]}{dt}$$

$$\boxed{\frac{dI_R}{dt} = \frac{1}{R}\cdot\frac{dV_c}{dt}} \quad \text{....(5)}$$

[∵ Resistance connected @ node (1). @ node 1 the potential is capacitive ∴ $I_R = \frac{V_c}{R}$]

sub (4) & (5) in equation (A)

$$\boxed{LC\cdot\frac{d^2V_c}{dt^2} + \frac{L}{R}\cdot\frac{dV_c}{dt} + V_c = V} \quad \text{— (6)}$$

$$\left[V_c = \frac{1}{c}\int I_c\cdot dt\right]$$

Taking Laplace Transform of equation no.6 we get

$$LC\left[s^2V_c(s) - sV_c(0) - \frac{dV_c(0)}{dt}\right] + \frac{L}{R}\left[sV_c(s) - V_c(0)\right]$$

$$+V_c(s) = \frac{V}{s} \quad \text{.......(7)}$$

Initial condition

$$\boxed{V_c(0) = 0} \quad ∴ \quad \boxed{\frac{dV_c(0)}{dt} = 0} \quad \text{.... (8)}$$

Substituting (8) in (7)

$$(7) \Rightarrow \boxed{LC\,s^2 V_C(s) + \frac{L}{R}\,s\,V_C(s) + V_C(s) = \frac{V}{s}} \quad \text{—— (9)}$$

Divide by LC on both sides of equation number (9)

$$s^2 V_C(s) + \frac{L}{R} \times \frac{1}{LC}\, s\,V_C(s) + \frac{1}{LC} V_C(s) = \frac{V}{sLC}$$

$$s^2 V_C(s) + \frac{1}{RC}\, s\,V_C(s) + \frac{1}{LC} V_C(s) = \frac{V}{sLC}$$

$$V_C(s)\left[s^2 + \frac{1}{RC}s + \frac{1}{LC}\right] = \frac{V}{sLC}$$

$$\therefore V_C(s) = \frac{\dfrac{V}{sLC}}{s^2 + \dfrac{1}{RC}s + \dfrac{1}{LC}}$$

$$\boxed{\therefore V_C(s) = \frac{V}{sLC\left[s^2 + \dfrac{1}{RC}s + \dfrac{1}{LC}\right]}} \quad \text{—— (10)}$$

* The natural frequency (or) the resonant frequency of damped oscillation is given by

$$\boxed{f_n = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{1}{4R^2C^2}}} \quad \text{—— (11)}$$

* For no transient oscillations all the roots of the equation should be real. one root is zero

i.e $\boxed{SLC = 0}$ (12) $\therefore \boxed{S = 0}$ (12) $\left[S = \frac{0}{LC} \right]$ $\therefore S = 0$

* for other two roots to be real, the roots of quadratic equation in the denominator should be real

$\therefore$ The roots of the quadratic equation $\left[s^2 + \frac{1}{RC}s + \frac{1}{LC}\right]$

$$s = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \left[a = 1;\ b = \frac{1}{RC};\ c = \frac{1}{LC}\right]$$

$$\therefore s = \frac{-\frac{1}{RC} \pm \sqrt{\left(\frac{1}{RC}\right)^2 - 4 \times 1 \times \frac{1}{LC}}}{2 \times 1}$$

$$s = -\frac{1}{2RC} \pm \sqrt{\frac{1}{4(RC)^2} - \frac{4}{4LC}}$$

$$\boxed{s = \frac{-1}{2RC} \pm \sqrt{\frac{1}{(2RC)^2} - \frac{1}{LC}}} \quad \text{— (12)}$$

* for the roots of the quadratic equation to be real the terms within the square root must be ≥ 0

Therefore $\frac{1}{(2RC)^2} - \frac{1}{LC} \geq 0 \rightarrow \frac{1}{(2RC)^2} \geq \frac{1}{LC}$

$$\Rightarrow \frac{1}{4R^2C^2} \geq \frac{1}{LC}$$

$$\Rightarrow R^2 \leq \frac{L\cancel{C}}{4C^{\cancel{2}}}$$

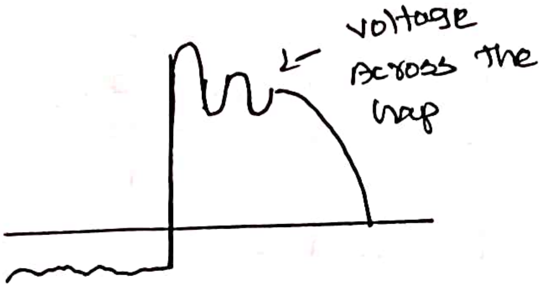
$$\Rightarrow R \leq \sqrt{\frac{L}{4C}}$$

$$\Rightarrow R \leq \sqrt{\frac{L}{4C}}$$

$$\Rightarrow \boxed{R \leq \frac{1}{2}\sqrt{\frac{L}{C}}}$$

Graph :- (oscillation)

(i) $R = \infty$



(ii) $R > \frac{1}{2}\sqrt{\frac{L}{C}}$

(iii) $R < \frac{1}{2}\sqrt{\frac{L}{C}}$

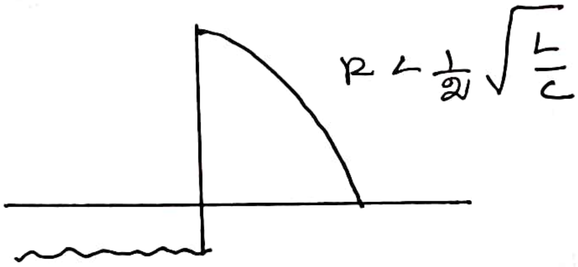
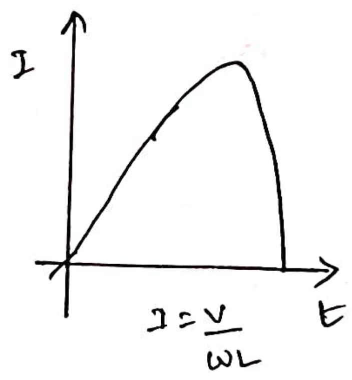


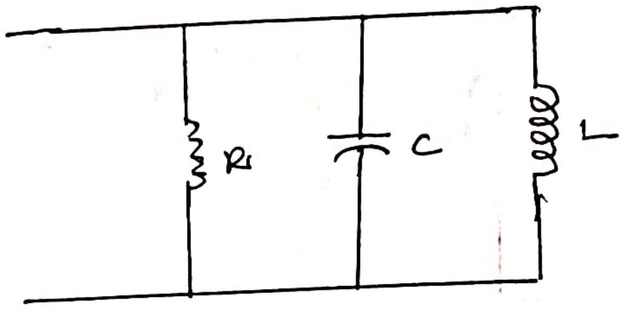
fig: Transient oscillations for different values of 'R'

* From the graph, if the value of Resistance 'R' is equal to or less than $0.5\sqrt{\frac{L}{C}}$ the oscillatory nature of the transient will not be there and rate of rise of restriking voltage will be within permissible limits of circuit breaker.

* For critical damping $\boxed{R = 0.5\sqrt{\frac{L}{C}}}$ is known as critical resistance.

Equivalent circuit for Resistance switching:-





* Viewing from the circuit Breaker contacts the circuit elements L and C appear in parallel. suppose that the fault current being interrupted is symmetrical. It is given by $\frac{V}{\omega L}$

* The infored current is $I = \left(\frac{V}{L}\right) * t$

where V → Instantaneous system voltage at the moment of interruptions.

Equivalent circuit for Interrupting the resistor current:-

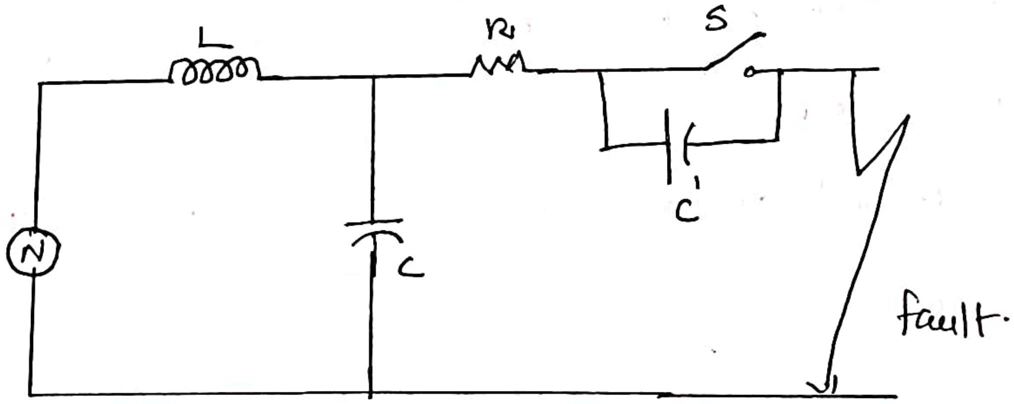


Fig:- Equ.ckt for Intr. fault current

where L → system inductance, C' → stray capacitance
R → resistor used to modify recovery transient
S → switch

(6)

* When the resistor current is subsequently interrupted a second transient will be initiated.
* To study its effect it is necessary to introduce the capacitance 'C' shunting the breaker.

1.4 LOAD SWITCHING:-

* The most frequent functions performed by some switching devices are due to switch on and switch off loads which in many cases instances can be represented by a parallel R-L circuit.

Circuit:-

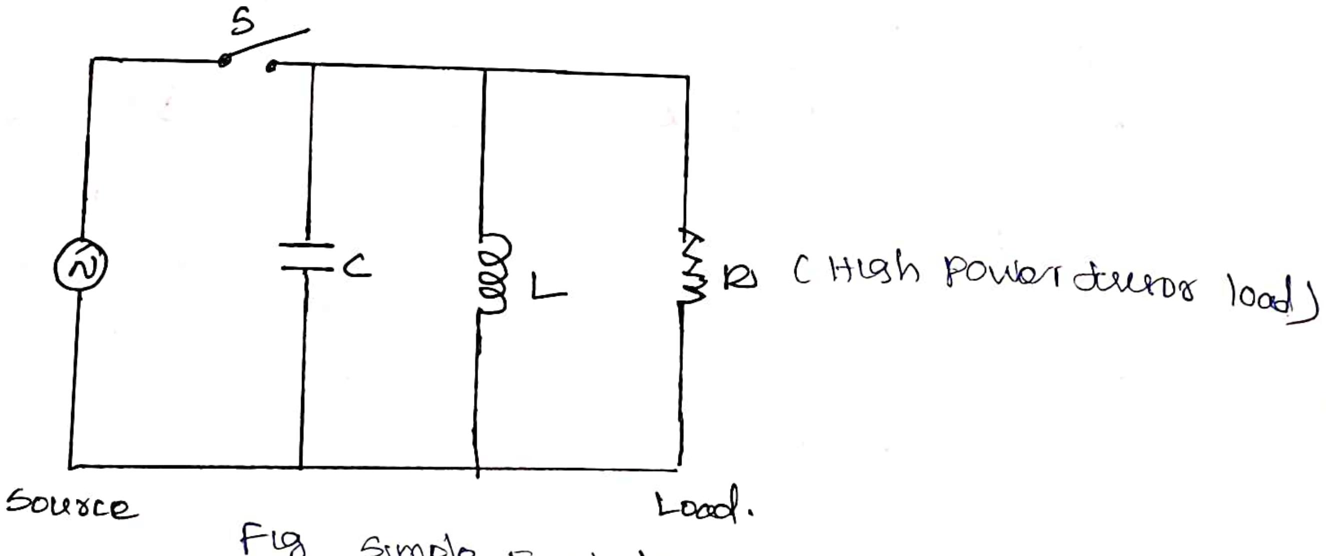


Fig Simple Equivalent Circuit.

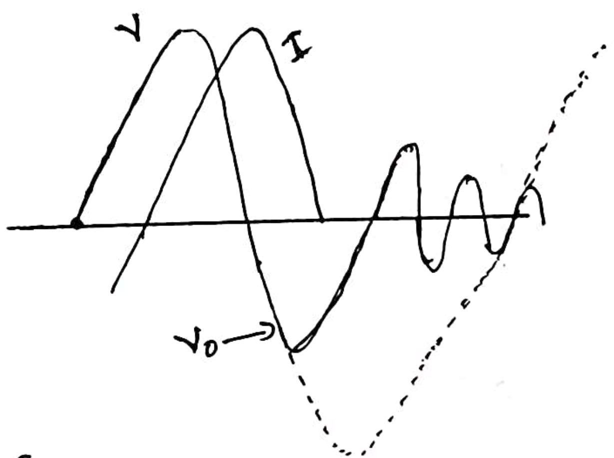
* Low power factor loads will be predominantly inductive whereas high power factor loads are will be predominantly resistive.

* when such load is switched off, the effective capacitance of load becomes important in determining the form of transient generated.

* The load depicted in the above fig. is has relatively high power factor.

* when the current extinguishes the instantaneous voltage and therefore the voltage across the load is Vo

* Now 'c' will be charged to this voltage and will subsequently discharge through 'L' and 'R'



(fig. b) Transient voltage across the load.

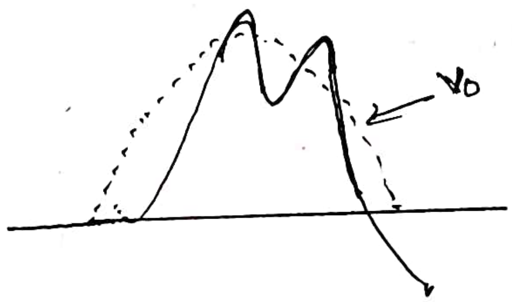


fig. c Transient voltage across the switch.

* In fig (b) This is shown as damped osciliatory discharge and is in fact a damped cosine wave.

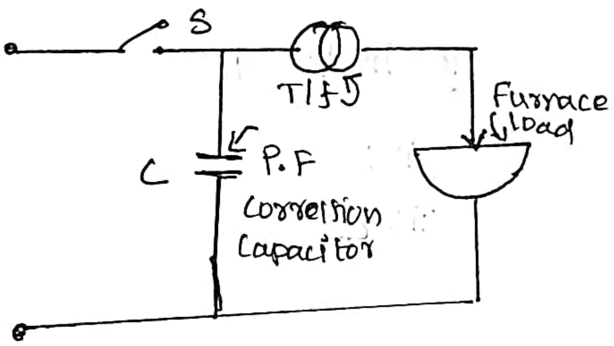
* The effect of power factor is interesting to observe as the power factor improves the current comes more and more into phase with the voltage so that Vo deminishes.

* At unity power factor (pure resistive load)

Voltage is zero when current is zero so there is no transient at all, the ~~st~~ situation is different if the power factor is corrected to unity.

Example:- (Arc furnace)

* Arc furnaces are common place in industry
* Ore is melted and metals are alloyed and refined by melting them with the intense heat of an electric arc, using graphic electrodes
* Such installations usually operate at low voltage and high current and are consequently fed by a step down furnace transformer.



Fig(a) Schematic representation of arc furnace.

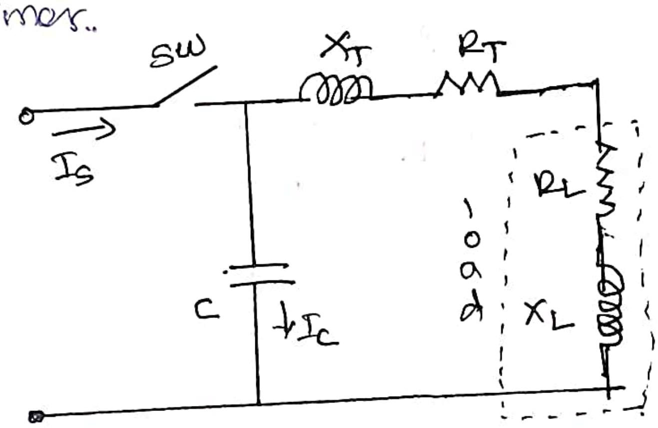


Fig.(b) Equivalent circuit of T/f. with load.

* They are characterized by low power factor and frequent switching.
* Capacitors are frequently connected to the high voltage bus to improve the power factor, they are switched with the transformer and furnace.

* The above fig shows the one phase of arc furnace an installation with its equivalent circuit.

* Delta and wye connections can be used, the figure shows one phase of a wye connected circuit.

Investigation of transient evoked by switching off the fully loaded transformer :-

The following details apply

Transformer: 60 Hz, 13.8 kV, 20 MVA Y/Y connected and solidly grounded.

* when fully loaded at rating power factor = 0.6. 'C' corrects the power factor to unity as seen at the supply bus.

Soln:- Given:-

$V_L = 13.8\ kV$

$P = 20\ MVA$

power factor $\cos\phi = 0.6$

To find:- L

(i) L

(ii) C

Soln:-

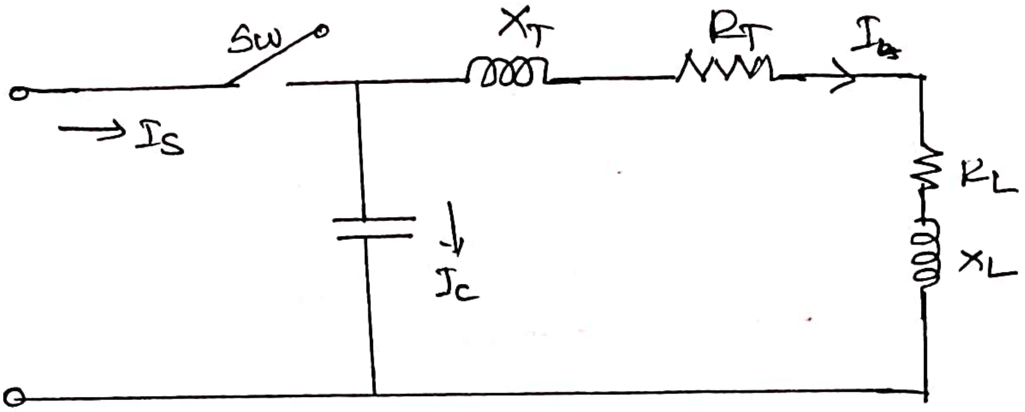
Load current $I_L = \dfrac{P}{V_{ph}}$

$V_L = \sqrt{3} V_{ph}$

$\left[\therefore V_{ph} = \dfrac{V_L}{\sqrt{3}}\right]$

$= \dfrac{20{,}000}{13.8 \times \sqrt{3}}$

$\boxed{I_L = 836.7\ A}$ —(1)

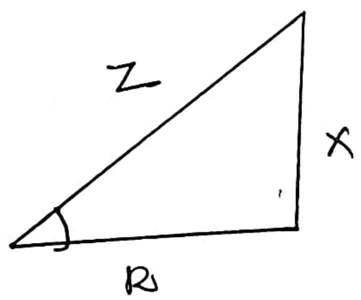


$$Z = \frac{V_{ph}}{I_{ph}} \qquad \left[V_{ph} = \frac{13.8}{\sqrt{3}} \right]$$

$$= \frac{13.8}{836.7 \times \sqrt{3}}$$

$$\boxed{Z = 9.522\,\Omega} \quad \text{—— (2)}$$

Impedance Triangle:-



Where $R \rightarrow$ Total - R

$X \rightarrow$ Total reactance

$$Z = R + jX$$

To find R

$R = $ $\cos\phi = \frac{R}{Z}$

$\therefore \; R = Z\cos\phi$

$R = 9.522 \cos\phi$

$R = 9.522 \times 0.6$

$R = 5.7132\Omega$

$\therefore \boxed{R = R_T + R_L = 5.7132\Omega}$ — 3

To find X:-

$\sin\phi = \dfrac{X}{Z}$

$X = Z \sin\phi$

$X = 9.522 \sin\phi$

$\cos\phi = 0.6$

$\therefore \phi = \cos^{-1}(0.6)$

$\phi = 53.13^{0}$

$\therefore X = 9.522 \sin(53.13^{0})$

$\boxed{X = 7.618\Omega}$ — 4 ie $\boxed{X = X_T + X_L = 7.618\Omega}$

To find L:-

w.k.T

$X_L = \omega L$

$L = \dfrac{X}{\omega}$

$= \dfrac{7.618}{2 \times \pi \times f}$

$\therefore L = \dfrac{7.618}{2 \times \pi \times 60}$ $\boxed{L = 20.2\,mH}$ — 5

→ when the current I_s is interrupted at current zero, the current I_c & I are equal and opposite

ie $I_c(0) = -I(0)$

$= 836.7 \sin\phi$

$$\boxed{I_c(0) = 669.4\ A} \quad \text{—— (6)}$$

W.K.T $V = I X_C$

$$X_C = \frac{V_{\cancel{ph}}}{I}$$

$$X_C = \frac{V_{mp}}{I_c}$$

$V_{mp} = \sqrt{2}\, V_{ph}$

$$X_C = \frac{\sqrt{2} \times V_{ph}}{\sqrt{3} \times I_c} \qquad \left[\because V_{ph} = \frac{V_L}{\sqrt{3}}\right]$$

Sub value of V_{ph} & I_c in above equation

$$X_C = \frac{\sqrt{2} \times 13.80 \times 10^3}{\sqrt{3} \times 669.4}$$

$$\boxed{X_C = 16.83\ \Omega} \quad \text{—— (7)}$$

W.K.T $X_{\cancel{C}}$

W.K.T $X_C = \frac{1}{\omega C}$

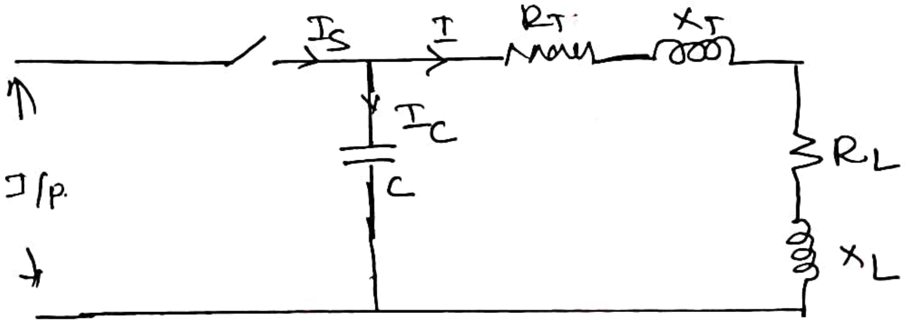
$\therefore C = \frac{1}{\omega X_C}$

$C = \frac{1}{2 \times \pi \times f \times X_C}$

$C = \frac{1}{2 \times \pi \times 60 \times 16.83}$

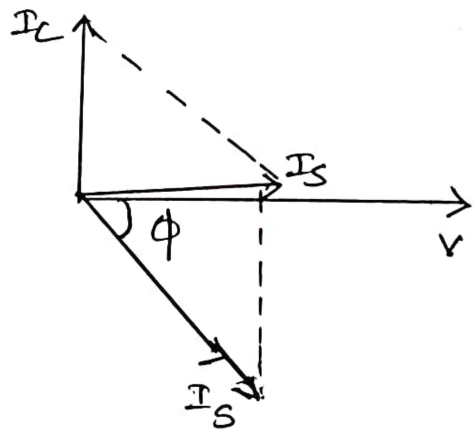
$\boxed{C = 157.6\ \mu F}$ — (8)

Steady state Condition - phasor diagram:-



From Fig:- $\bar{I}_S = \bar{I} + \bar{I}_C$

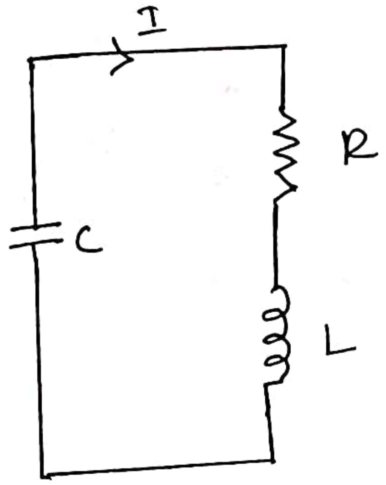
$I \rightarrow$ lags applied voltage ; $I_C \rightarrow$ leads the applied voltage



[Fig: phasor diagram

Post Interrupt Transient:-

After opening of C.B the circuit reduces to



$[\because R = R_T + R_L$

Apply KVL

$$V_R + V_L + V_C = 0$$

$$\therefore \quad IR + L.\frac{dI}{dt} + \frac{1}{C}\int I.dt = 0 \quad — (9)$$

Diff eqn (9) w.r. to 't'

$$R.\frac{dI}{dt} + L.\frac{d^2I}{dt^2} + \frac{I}{C} = 0 \quad — (10)$$

$\div$ by L & Re arrange

$$\frac{d^2I}{dt^2} + \frac{R}{L}.\frac{dI}{dt} + \frac{I}{LC} = 0 \quad — (11)$$

$$\frac{d^2I}{dt^2} + \frac{1}{T_s}.\frac{dI}{dt} + \frac{I}{T_e^2} \quad — (12)$$

Where $T_s \rightarrow$ Series Time constant

$T \rightarrow$ Total circuit Time constant

Note:

[ie $T_s = \frac{L}{R}$

W.K.T $T_p = RC$

$T_s \cdot T_p = \frac{L}{\cancel{R}} \times \cancel{R}C$

$T_s \cdot T_p = LC$

If $T_s = T_p = T$

$T \cdot T = LC$

$T^2 = LC$]

(12) $\Rightarrow$ $$\frac{d^2I}{dt^2} + \frac{1}{T_s} \cdot \frac{dI}{dt} + \frac{I}{T^2} = 0$$

Taking Laplace transform of equation no. (12)

$$\left[s^2 I(s) - sI(0) - \frac{dI(0)}{dt}\right] + \frac{1}{T_s}\left[sI(s) - I(0)\right] + \frac{I(s)}{T^2} = 0$$

Rearranging the above equation

$$s^2 I(s) - sI(0) - \frac{dI(0)}{dt} + \frac{sI(s)}{T_s} - \frac{I(0)}{T_s} + \frac{I(s)}{T^2} = 0$$

$$I(s)\left[s^2 + \frac{s}{T_s} + \frac{1}{T^2}\right] - I(0)\left[s + \frac{1}{T_s}\right] - \frac{dI(0)}{dt} = 0$$

$$I(s)\left[s^2 + \frac{s}{T_s} + \frac{1}{T^2}\right] = \left[s + \frac{1}{T_s}\right]I(0) + \frac{dI(0)}{dt}$$

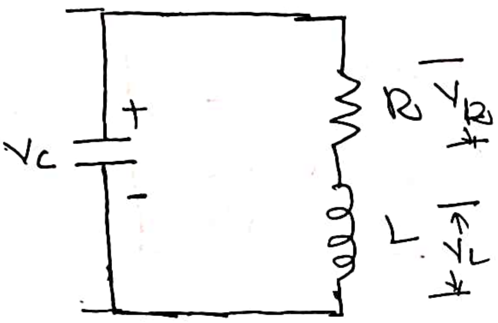
$$\boxed{I(s)\left[s^2 + \frac{s}{T_s} + \frac{1}{T^2}\right] = \left[s + \frac{1}{T_s}\right]I(0) + I'(0)} \quad \text{--- (13)}$$

Note :- Transform

$$L\left[\frac{d^2y}{dt^2}\right] = \left[s^2 y(s) - s y(0) - \frac{dy(0)}{dt}\right]$$

$$L\left[\frac{dy}{dt}\right] = \left[s y(s) - y(0)\right]$$

To find $I'(0)$:-



$$V_c = V_R + V_L$$

$$V_c = IR + L \cdot \frac{dI}{dt}$$

$$V_c(0) = I(0)R + L \cdot \frac{dI(0)}{dt}$$

$\left[V_c(0) = 0 \right.$

$\left. \frac{dI(0)}{dt} = I'(0) \right]$

$$I(0)R + L \cdot I'(0) = 0$$

$$L \cdot I'(0) = -I(0)R$$

$$I'(0) = -\frac{I(0)R}{L}$$

$\left[T_s = \frac{L}{R} \right]$

$$\boxed{\therefore I'(0) = \frac{-I(0)}{T_s}} \quad \text{(14)}$$

Sub (14) in (13)

$$\therefore I(s)\left[s^2+\frac{s}{Ts}+\frac{1}{T^2}\right]=\left[s+\frac{1}{Ts}\right]I(0)-\frac{I(0)}{Ts}$$

$$\therefore I(s)\left[s^2+\frac{s}{Ts}+\frac{1}{T^2}\right]=s\,I(0)+\frac{I(0)}{Ts}-\frac{I(0)}{Ts}$$

$$I(s)\left[s^2+\frac{s}{Ts}+\frac{1}{T^2}\right]=s\,I(0)$$

$$\boxed{I(s)=\frac{s\,I(0)}{s^2+\frac{s}{Ts}+\frac{1}{T^2}}} \quad \text{—(15)}$$

For series circuit $\lambda=\frac{1}{\eta}$ $\quad\quad \eta=\frac{R}{Z_0}$

$$\therefore \boxed{\lambda=\frac{Z_0}{R}} \quad \text{—(16)} \quad\quad Z_0=\sqrt{\frac{L}{C}}$$

$$Z_0=\sqrt{\frac{20.2\times10^{-3}}{157.6\times10^{-6}}}$$

$$\boxed{Z_0=11.43\,\Omega}$$

$$\boxed{R=5.713\,\Omega} \quad \text{from (3)}$$

$$\therefore \lambda=\frac{11.43}{5.713} \quad\quad \boxed{\lambda=2} \quad \text{—(17)}$$

* From the η curve the current starting at $-669.4A$ swings to a positive peak slightly in excess of half of this value a half cycle later

and then to about -0.23×669.4 after a further half cycle

To compute Transformer terminal voltage

$$\boxed{\frac{d^2 v_c}{dt^2} + \frac{1}{Ts} \cdot \frac{dv_c}{dt} + \frac{v_c}{T^2} = 0} \quad \text{— (18)}$$

Taking laplace transform of above equation

$$V_c(s)\left[s^2 + \frac{s}{Ts} + \frac{1}{T^2}\right] = \left[s + \frac{1}{Ts}\right] V_c(0) + V_c'(0)$$

$$\boxed{V_c'(0) = -\frac{I(0)}{C}}$$

$$\left[\; C \cdot \frac{dv_c}{dt} = -I \qquad V_c' = -\frac{I(0)}{C} \;\right]$$

L(19)

$$\boxed{V_c(s) = \frac{1}{s^2 + \frac{s}{Ts} + \frac{1}{T^2}} \cdot \frac{I(0)}{C}} \quad \text{— (19)}$$

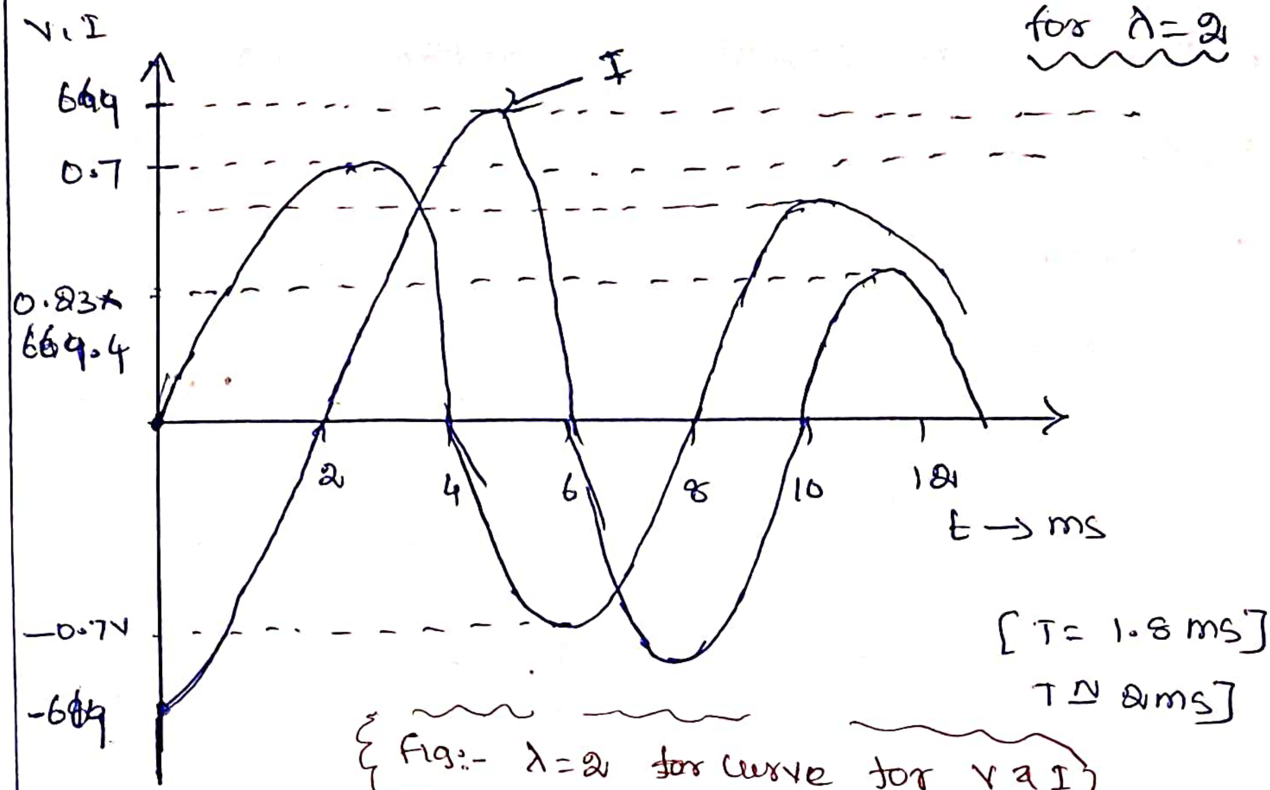
$$T^2 = LC$$

$$T = \sqrt{LC}$$

$$T = \sqrt{20.2 \times 10^{-3} \times 157.6 \times 10^{-6}}$$

$$\boxed{T = 1.784\ ms} \quad \text{— (20)}$$

for $\lambda = 2$



{Fig:- $\lambda = 2$ for curve for V & I}

* The first peak voltage reaches about 72% of undamped value

* The first voltage after current zero is therefore

$$\boxed{0.72 \times 669.4 \times 11.43 = 5.509 \text{ V}}$$

1.5 Normal and Abnormal switching Transients:-

1.5.1 Normal switching Transients:-

"When a switch opens in a single phase circuit (or) when a discharged capacitor energized at the peak of system voltage cycle and that power frequency current attains twice its normal peak rms value" then these transients are called as "Normal switching Transients".

1.5.2 Abnormal switching Transients:-

"If the voltages and currents far in Excess of twice its normal peak rms value, then such transients are called as abnormal switching Transients".

Ex:- (i) Bus transfer switching operations

(ii) Inception and clearing of system faults.

(iii)

The mechanism that occurred frequently are

(a) current chopping (b) Restriking switch

(c) Lightning induced transients.

(A) Current Chopping:-

Current chopping is the name given to the rapid current reductions prior to the natural current zero of the power system which fuses (or) circuit Breaker can force when clearing a fault.

Ex:- An unloaded Transformer

* When there is inductance in the circuit, this rapid current can produce high over voltage nearly 10 times the normal circuit voltage.

Ex:- Unloaded Transformer

(b) Restriking of switch:-

* capacitor switching can be trouble some of if the switch restrikes after current interruption the capacitor voltage remains nearly constant at maximum system voltage, since the interruption occured at current zero which is 90^0 apart from the voltage zero, while the system follows the normal sinewave.

Ex:- An ungrounded power system experiences an arcing ground fault.

(c) Lightning - Induced Transients:-

* Lightning induced transients are even less predictable because there is a wide range of coupling possibilities.

* Protection against lightning effects includes two categories

(a) Direct Effect - Tower line, tallest Buildings etc.,

(b) Indirect Effects - Induced voltage is nearby Electrical and Electronic systems

1.6 CURRENT SUPPRESSION:-

"when an Abnormal conditions occurs, in the power system, the current carried by a power switch does not normally cease when its contacts separate, and continues to flow through an arc until its reaches its periodic zero.

* Arc suppression devices in the circuit breaker bring the current to zero abruptly and prematurely ahead of the normal current zero.

* This phenomenon is referred to as current chopping and is an Example of what is known generically as current suppression."

Circuit:- (Equivalent Circuit)

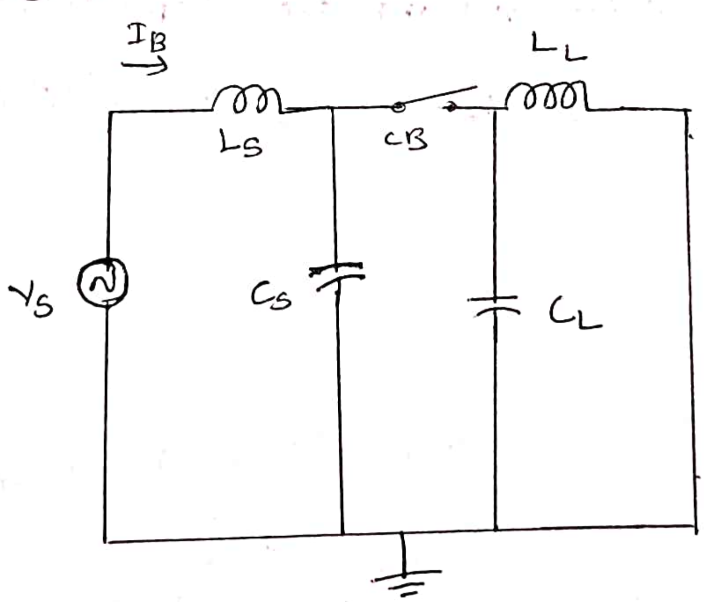


Fig:- Equivalent Circuit for typical current suppression.

* At the instant when current suppression occurs, the energy stored in the load inductance is transferred to the load side capacitance and thus creating a condition where overvoltages can be generated.

* The voltage and current relationships are illustrated in following graph.

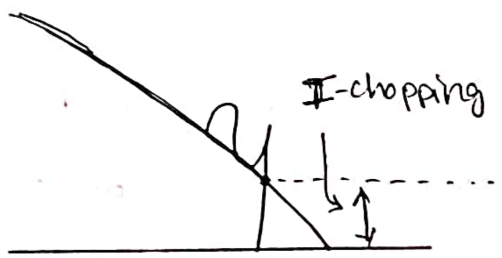


Fig (a) chopped current across C-B

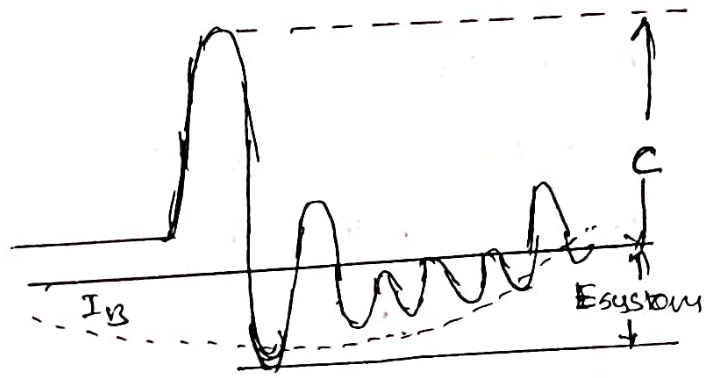


fig.b:- Transient voltage across the circuit Breaker.

* During this process the energy stored in the inductor $\frac{1}{2}Li^2$ will be discharged into the capacitor, so that energy the capacitor will be charged to a 'V' called prospective voltage ie $[\frac{1}{2}CV^2]$

$$\cancel{\frac{1}{2}}Li^2 = \cancel{\frac{1}{2}}CV^2$$

$$\therefore V^2 = i^2 \frac{L}{C}$$

$$\boxed{V = i\sqrt{\frac{L}{C}}}$$

& The surge Impedance

$$\frac{V^2}{i^2} = \frac{L}{C}$$

$$\boxed{\therefore Z = \sqrt{\frac{L}{C}}}$$

* This prospective voltage is extremely high as compared to the normal system voltage.

* Frequency of oscillation is given by

$$\boxed{f_n = \frac{1}{2\pi\sqrt{LC}}}$$

* ~~To examine the problem, the following example has been~~

* The following example illustrates the severity of current suppression problem.

Example:-

Consider a 1000KVA, 13.8KV transformer of the kind found in substation of industrial plants, the magnetizing current is typically 1.5 A Thus.

Given: Voltage = 13.8KV

Power = 1000KVA

current I_0 = 1.5 A   frequency = 60HZ

To find:-

(i) $V = i\sqrt{\frac{L}{C}}$

* $L_m = \frac{V_{ph}}{\omega I} = \frac{13.8 \times 10^3}{\sqrt{3} \times 2\times\pi\times 60 \times 1.5}$

$V_L = \sqrt{3}V_{ph}$

$\left[V_{ph} = \frac{V_L}{\sqrt{3}}\right]$

$\boxed{L_m = 14H}$

* The Effective capacitance will vary depending on the type of winding and the insulation whether oil, air, or some other material, but would be in the range 1000-7000pF

* suppose we choose 5000PF then

$\boxed{C = 5000PF}$ then

$$Z = \sqrt{\frac{L}{C}}$$

$$= \sqrt{\frac{14}{5000 \times 10^{-12}}}$$

$$\boxed{Z = 52915\,\Omega}$$

Now:- ~~The~~ If the circuit breaker chops the peak current which because of harmonic distortion might be $\boxed{2.5A}$

Now $V = i\sqrt{\frac{L}{C}}$ $\left[\because Z = \sqrt{\frac{L}{C}}\right]$

$\therefore V = iZ$

$= 2.5 \times 52915$

$\therefore V = 132.287\ kV$

$$\boxed{\therefore V \simeq 132\,kV}$$

* This is indeed an abnormal overvoltage for a 13.8kV system.

1.6.1 Analysis of current chopping in unloaded Transformer:-

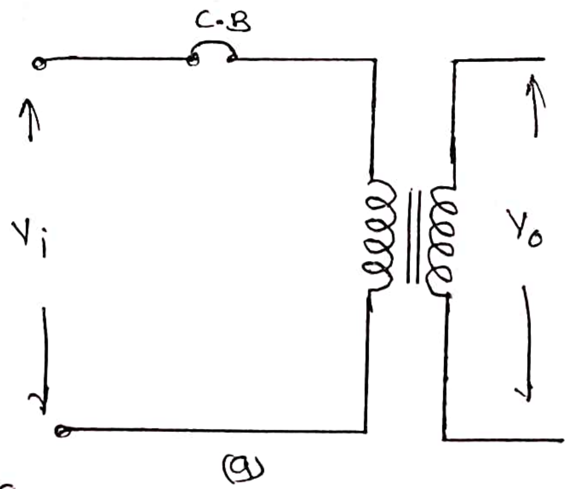
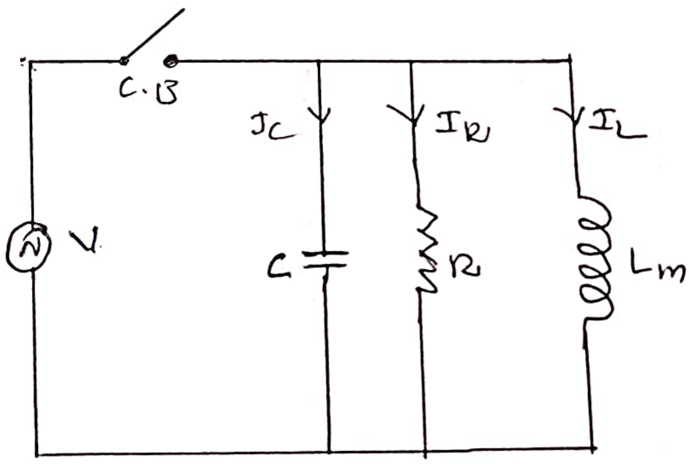


fig:-(a) C.B & unloaded T/f



fig(b) Equivalent circuit.

* Let I_C be the current through the capacitor, I_R be the current through the resistor, and I_L be the current through the inductor

* Let the value of current chopped be I_0

* Immediately after the current zero chop occurs there is no path through the switch, so that thereafter the sum of the currents in the three branches must be zero.

$$\boxed{I_R + I_L + I_C = 0} \quad \text{--- (1)}$$

where
$$\left.\begin{aligned} I_R &= \frac{V}{R} \\ I_L &= \frac{1}{L}\int V \cdot dt \\ I_C &= C \cdot \frac{dV}{dt} \end{aligned}\right\} \text{--- (2)}$$

sub (2) in (1)

$$\frac{V}{R} + \frac{1}{L}\int V \cdot dt + C \cdot \frac{dV}{dt} = 0 \quad \text{--- (3)}$$

Differentiate equation no. (3) w.r.to 't'

$$\frac{1}{R} \cdot \frac{dV}{dt} + C \cdot \frac{d^2V}{dt^2} + \frac{V}{L} = 0 \quad \text{...... (4)}$$

Rearranging

$$C\frac{d^2V}{dt^2} + \frac{1}{R} \cdot \frac{dV}{dt} + \frac{V}{L} = 0 \quad \text{...... (5)}$$

÷ above Equation by 'c'

$$\boxed{\frac{d^2v}{dt^2} + \frac{1}{RC}\cdot\frac{dv}{dt} + \frac{v}{LC} = 0} \quad \text{------ (6)}$$

Taking Laplace transform on both the sides of above equation

$$\left[s^2 V(s) - sV(0) - \frac{dV(0)}{dt}\right] + \frac{1}{RC}\left[sV(s) - V(0)\right] + \frac{V(s)}{LC} = 0$$

$$V(s)\left[s^2 + \frac{1}{RC}s + \frac{1}{LC}\right] - sV(0) - \frac{V(0)}{RC} - \frac{dV(0)}{dt} = 0$$

$$\boxed{V(s)\left[s^2 + \frac{1}{RC}s + \frac{1}{LC}\right] = \left[s + \frac{1}{RC}\right]V(0) + V'(0)} \quad \left[\because \frac{dV(0)}{dt} = V'(0)\right]$$

— (7)

$V(0) \rightarrow$ voltage across the circuit breaker contacts at the instant of current stop.

$V'(0) \rightarrow$ Rate of change of voltage across the circuit breaker contacts at the instant of current chop.

$$V'(0) = \quad \therefore \; C\cdot\frac{dV}{dt} = -I_c$$

$$\frac{dV(0)}{dt} = \frac{-I_c(0)}{C}$$

$$\therefore \; V'(0) = \frac{-I_c(0)}{C} \qquad [I_c(0) = I_0]$$

$$\boxed{V'(0) = \frac{-I_0}{C}} \quad \text{— (8)}$$

since at the moment the switch chops the chopped current must be diverted into the capacitor. so combining equations (7) & (8)

$$\boxed{V(s) = \underbrace{\frac{s\,V(0)}{\left[s^2+\frac{s}{RC}+\frac{1}{LC}\right]}}_{\mathrm{I}} + \underbrace{\frac{V(0)/RC}{s^2+\frac{s}{RC}+\frac{1}{LC}}}_{\mathrm{II}} - \underbrace{\frac{I_0}{C\left[s^2+\frac{s}{RC}+\frac{1}{LC}\right]}}_{\mathrm{III}}}$$

* The first and second term that represents that the normal transient that would occur is the transformer were disconnected from the supply, with no current chopping

* The third term represents direct consequence of current chop and is the one potentially capable of creating an abnormal over voltage.

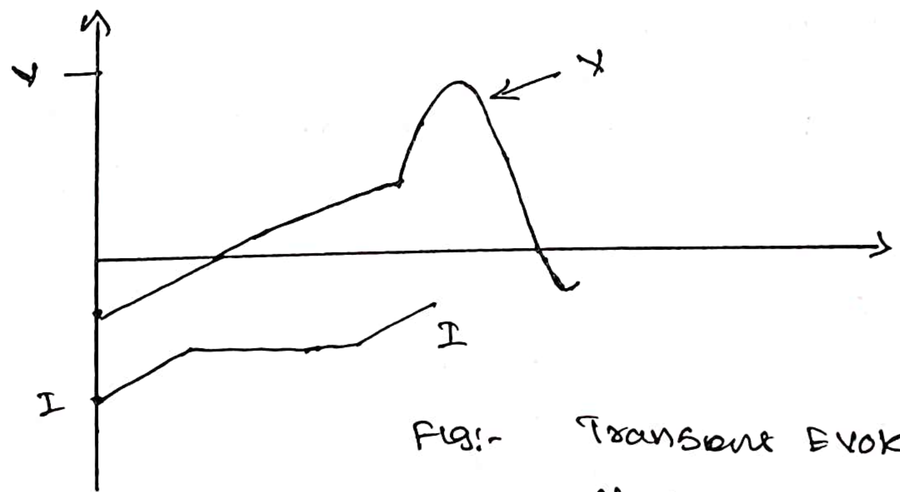


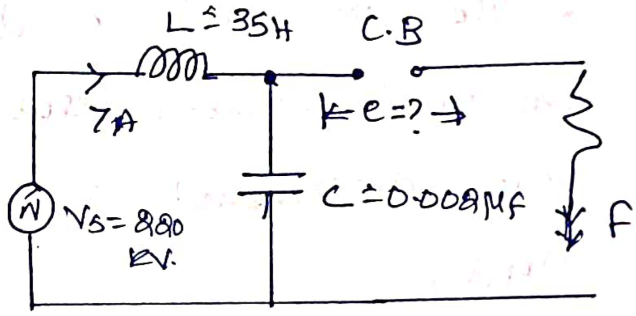
Fig:- Transient Evoked by the chop.

NUMERICAL PROBLEM - CURRENT CHOPPING

Example 1:

A 220 kV circuit breaker interrupting a magnetizing current of 10 A rms of transformer. Let the current be chopped at a instantaneous value of 7A. Let the value of inductance and capacitance be 35H and 0.002 μF. Determine the value of voltage appearing across a pole of circuit breaker.

Given:-



Voltage = 220 kV

$L = 35H$

$C = 0.002 \mu F$

Instantaneous current (i) = 7A

To find:-

(i) Voltage appearing across the pole of C.B

Formula to be used:-

(1) $e = i\sqrt{\frac{L}{C}}$ where $i \rightarrow$ Instantaneous current

Soln:-

$$e = i\sqrt{\frac{L}{C}}$$

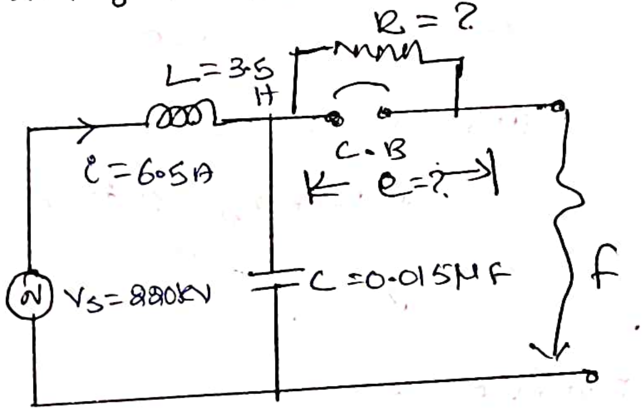
$$e = 7\sqrt{\frac{35}{0.002 \times 10^{-6}}}$$

$\boxed{e = 926 kV}$ → Voltage appears across the pole of C.B

Example 2:-

In a system having 220 kV the line to ground capacitance 0.015 μF, inductance 3.5 H. Determine the voltage appearing across pole of circuit breaker if a magnetizing current of 6.5 A instantaneous is interrupted. Determine also the value of resistance to be used across the contacts to eliminate the restriking voltage.

Given data:-



$V = 220\,kV$

$C = 0.015\,\mu F$

$L = 3.5\,H$

$i = 6.5\,A$ (instantaneous)

To find:-

(i) e → Voltage across pole of C.B

(ii) Resistance (R) – To eliminate restriking voltage

Formula to be used:-

(i) $e = i\sqrt{\frac{L}{C}}$

(ii) $R = 0.5\sqrt{\frac{L}{C}}$

Soln:-

(i) $e = i\sqrt{\frac{L}{C}}$ – voltage across pole of C.B

$= 6.5\sqrt{\frac{3.5}{0.015\times10^{-6}}}$

$\boxed{e = 99.3\,kV}$

ii) Resistance to be used to eliminate restriking voltage:-

$$R = 0.5\sqrt{\frac{L}{C}}$$

$$R = 0.5\sqrt{\frac{3.5}{(0.015 \times 10^{-6}}}$$

$$\boxed{R = 7.637 K\Omega}$$

Result:-

(i) voltage appear across the pole of C.B $\boxed{e = 99.3 kV}$

(ii) Resistance to be used to eliminate the restriking voltage $\boxed{R = 7.637 K\Omega}$

Example 3:-

In 132KV transmission system the phase to ground capacitance is 0.01μF. The inductance being 6H. Calculate the voltage being appearing across ~~a~~ the pole of circuit breaker is a magnetizing current of 10A is interrupted Find the value of resistance to be used across contact space to eliminate the striking voltage transient.

Given data:-

$L = 6\ H$

$C = 0.01\mu F$

$i = 10\ A$

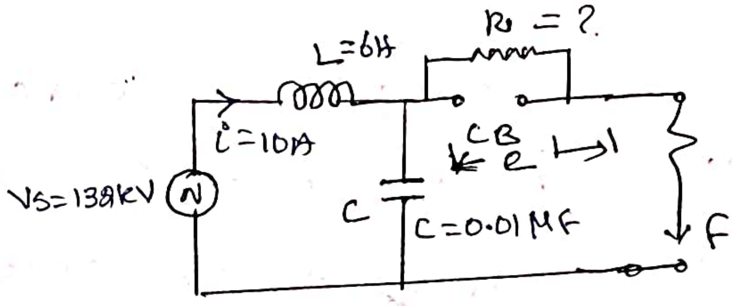
$V = 132\ KV$

To find:-

(i) e - voltage appearing across contacts of C.B

(ii) Resistance (R) - To eliminate restriking voltage

Formula to be used:-



(i) $e = i\sqrt{\frac{L}{C}}$

(ii) $R = 0.5\sqrt{\frac{L}{C}}$

soln:-

(i) Voltage appearing across contacts of C.B:-

$$e = 10\sqrt{\frac{6}{0.01\times10^{-6}}}$$

$$\boxed{e = 245kV}$$

(ii) value of resistance to be used across contacts of C.B:-

$$R = 0.5\sqrt{\frac{L}{C}}$$

$$R = 0.5\sqrt{\frac{6}{0.01\times10^{-6}}}$$

$$\boxed{R = 12.24k\Omega}$$

Result:-

(i) voltage appearing across contacts of C.B $\boxed{e = 245kV}$

(ii) value of 'R' to be used across contacts of C.B $\boxed{R = 12.24k\Omega}$

1.7 CAPACITANCE SWITCHING:-

* Another cause of excessive voltage transients across the circuit breaker contacts is the interruption of capacitive currents.

" Shunt capacitors are employed to correct a lagging power factor and in some cases to provide voltage support for the system. This is called capacitance switching."

Ex:- (i) Energizing and De-energizing of capacitor banks

(ii) Dropping of overhead lines (or) underground cables

(iii) Frequent switching in and out frequently as the system load varies and the system fluctuates.

Magnitude of capacitive currents encountered in practice are:-

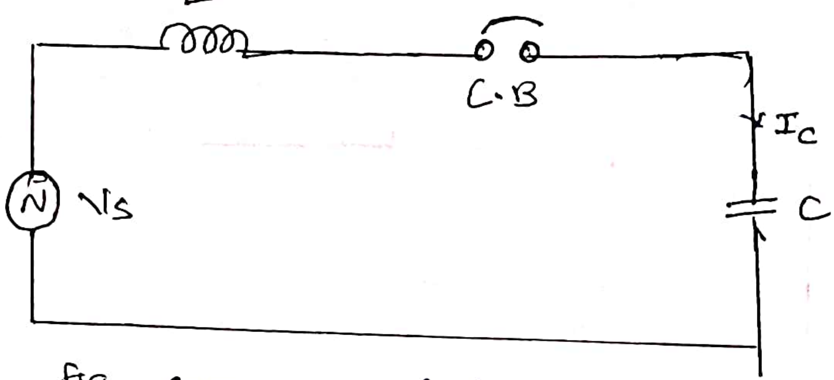
(a) Unloaded lines – charging currents upto 10 A

(b) underground cables – charging currents upto 100 A

(c) Capacitor banks – currents upto 1400 A

1.7.1 Equivalent circuit of an unloaded transmission line:-

Consider the simple equivalent circuit of an unloaded transmission line shown in fig.



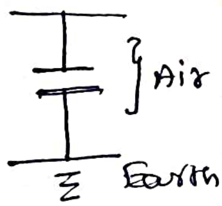


fig. capacitor switching

* Though the line is unloaded, will actually carry a capacitive current (i) on account of appreciable amount of capacitance 'c' between line and earth.

* When a capacitor is connected to the system, the leading current that it draws, flowing through the inductance of the system, causes the capacitor voltage to be somewhat greater than that at the open circuit system voltage.

→ This condition is called 'Ferranti rise' or "negative regulation."

* Because of relative phase of current and voltage (i.e the current leads the voltage by approximately 90°) the capacitor is charged to a maximum voltage when the switch interrupts.

The first waveform Fig(a) shows the relative phase difference between the voltage and current.

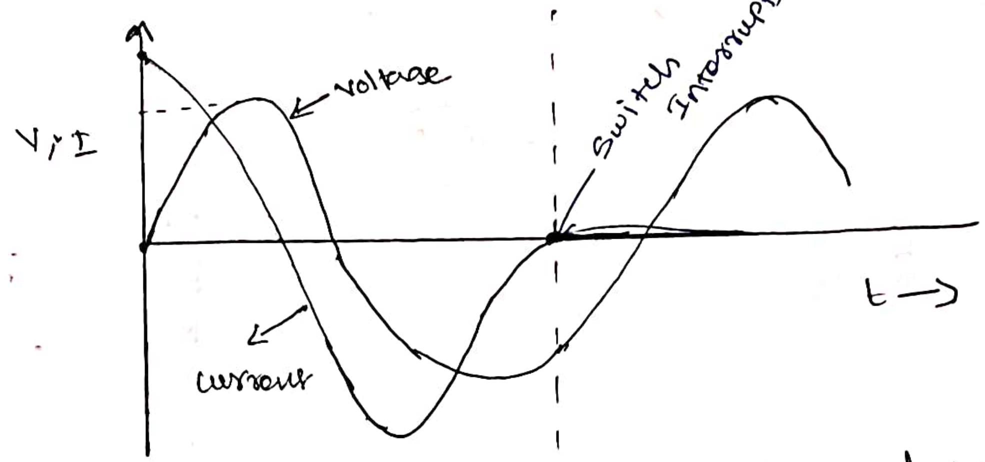
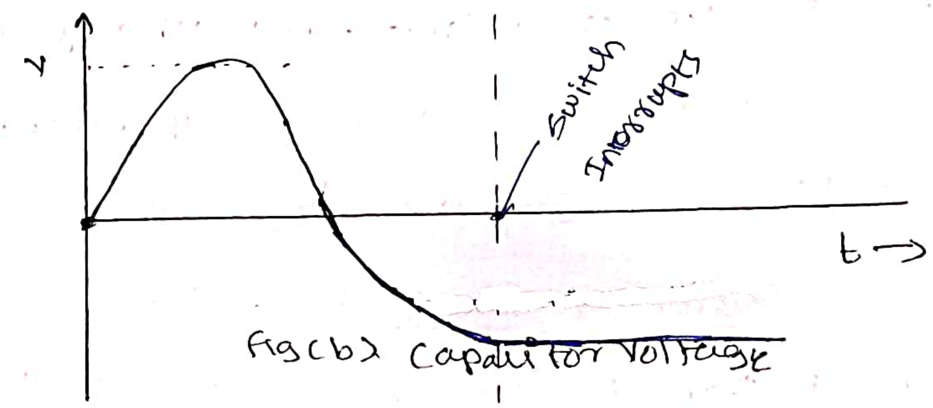
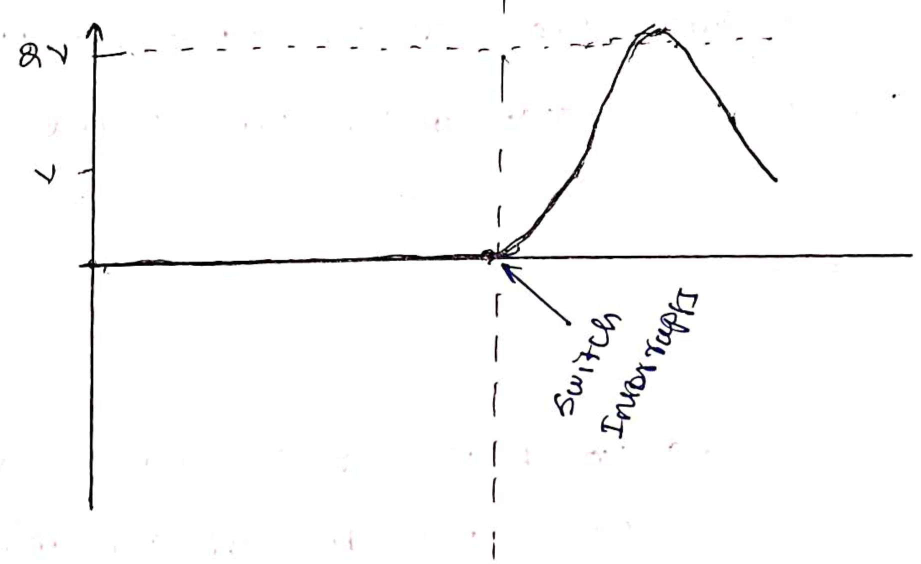


Fig (a). System voltage and current.



Fig(b) Capacitor Voltage



Fig(C) (Voltage across the switch)

* When switch interrupts the capacitance now isolated from the source retains its charge as shown in fig.(b)

* As a consequence of this charge which is present in the capacitance, that half a cycle later after current zero the voltage across a switch reaches a peak value of '2V' which is potentially dangerous, and it can be seen from ~~fig~~ figure 'c'.

1.7.2) Capacitance switching - showing the Effect of source regulation:-

* When the capacitor is disconnected from the source, the potential of source side C.B will return to the lower value after some oscillations

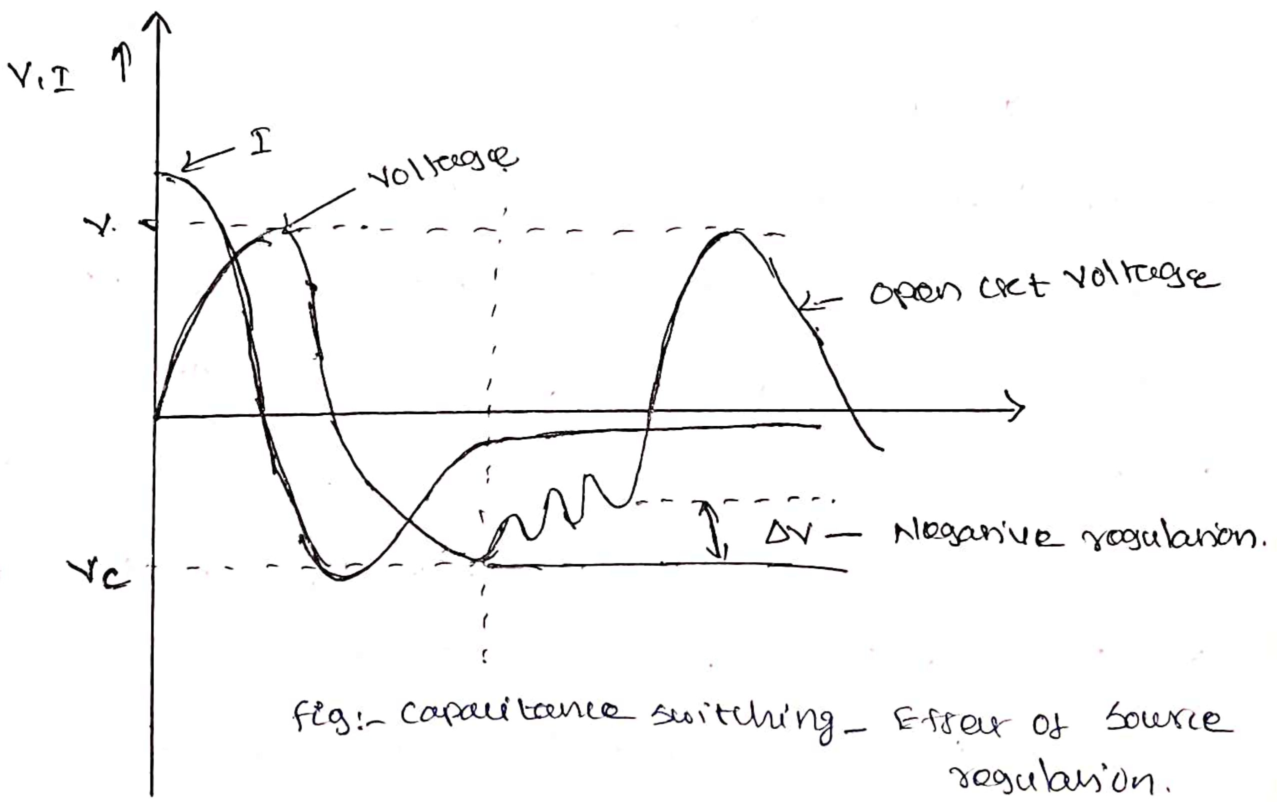
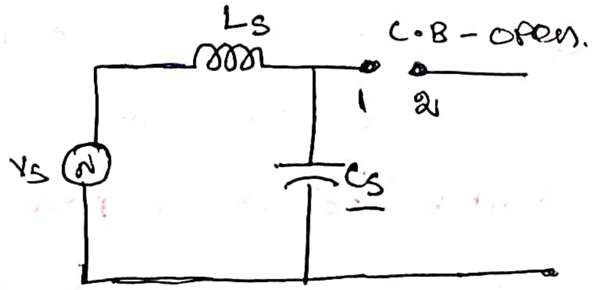


fig:- capacitance switching - Effect of source regulation.

* The oscillations are produced due to presence of source inductance and stray capacitance adjacent to the breaker on the source side.



$L_s \rightarrow$ Source inductance

$C_s \rightarrow$ Stray capacitance

Fig:- After C.B open - Source side

→ ΔV in wave form represent the negative regulation and it can be ~~elim~~ eliminated for relatively weak systems.

1.7.3 Capacitance switching - with a ~~St~~ Restrike:-

* Problem arises when the switching operation is unsuccessful and leads to restrike (or) reignites in the course of opening.

* The ~~...~~ event is termed reignition if the switch breaks down and current conduction ~~...~~ re-established within ~~...~~ half a cycle of current interruption.

* If the breakdown occurs later it is called "restrike"

* Some circuit breakers when called upon to interrupt a load or fault current, do not do so at the first current zero and waits for some time and leads to restrike ~~...~~ voltage.

* Consider the restrike takes place when the voltage reaches its peak. (2V)

* This is an L-C circuit. So due to this sudden disturbance, the voltage across the capacitor is subject to oscillation.

* The frequency of such oscillation is given by

$$f_0 = \frac{\omega_0}{2\pi}$$

$$\boxed{f_0 = \frac{1}{2\pi\sqrt{LC}}}$$

where L → inductance of the supply

C → Capacitance of the bank.

* Restrike current will be the instantaneous voltage across the switch divided by the circuit surge Impedance.

* Restriking current = $\dfrac{\text{Instantaneous Voltage } [2V_p]}{\text{surge Impedance } (Z)}$

$$= \frac{2V_p}{\left[\frac{L}{C}\right]^{1/2}} \sin \omega t \qquad \left[Z = \sqrt{\frac{L}{C}} \right]$$

$$\boxed{I_{RS} = 2V_p \left[\frac{C}{L}\right]^{1/2} \sin \omega t}$$

wave form:-

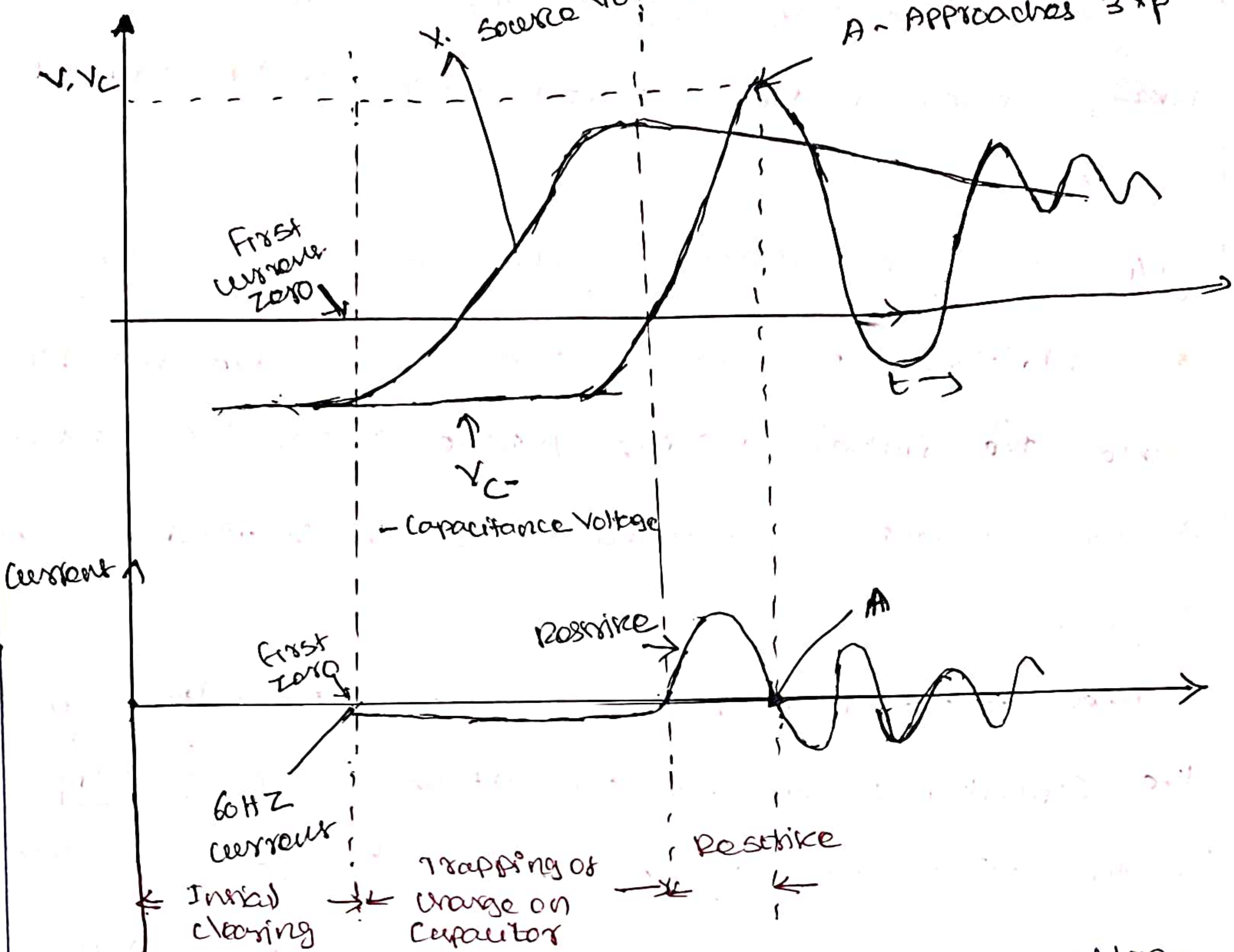


Fig:- Capacitive switching - strike peak voltage.

* The above fig. shows the initial clearing, trapping of charge on the capacitor and the subsequent restrike.

* when the C.B interrupts the current at point 'A', the voltage across the capacitor is high approximately 3 times the peak value ~~of~~ Vp. (3Vp)

* The transient voltage excursion to $3V_p$ is an abnormal over voltage and is the consequence of Energy stored in the capacitor bank at the time of restrike.

1.7.4 Capacitance switching with multiple restrikes

* Additional restrikes of the switch are possible since ~~the~~ the initial recovery voltage across the switch is again quite low following interruption of first restrike current.

* when at the instant of re-ignition the voltage on the capacitor 'c' was $-V$, the voltage is then $\boxed{+3V_p}$ at the first zero crossing of the transient current.

* The recovery voltage across the contact gap has now increased to $\boxed{4V_p}$

* Another re-ignition is likely to occur in the arcing contacts of the circuit breaker.

* Now the voltage on capacitor 'c' has increased to $5V_p$ and the voltage across the breaker contacts is now $\boxed{6V_p}$

* when a couple of re-ignition occur in this way called "multiple restrike"

* ~~So~~, So, very high voltages builds up across the interrupting chamber, and it is most likely that a flash-over takes place on the outside chamber of the interrupter.
* The circuit breaker is short circuited out of the system in this way and cannot function any more.

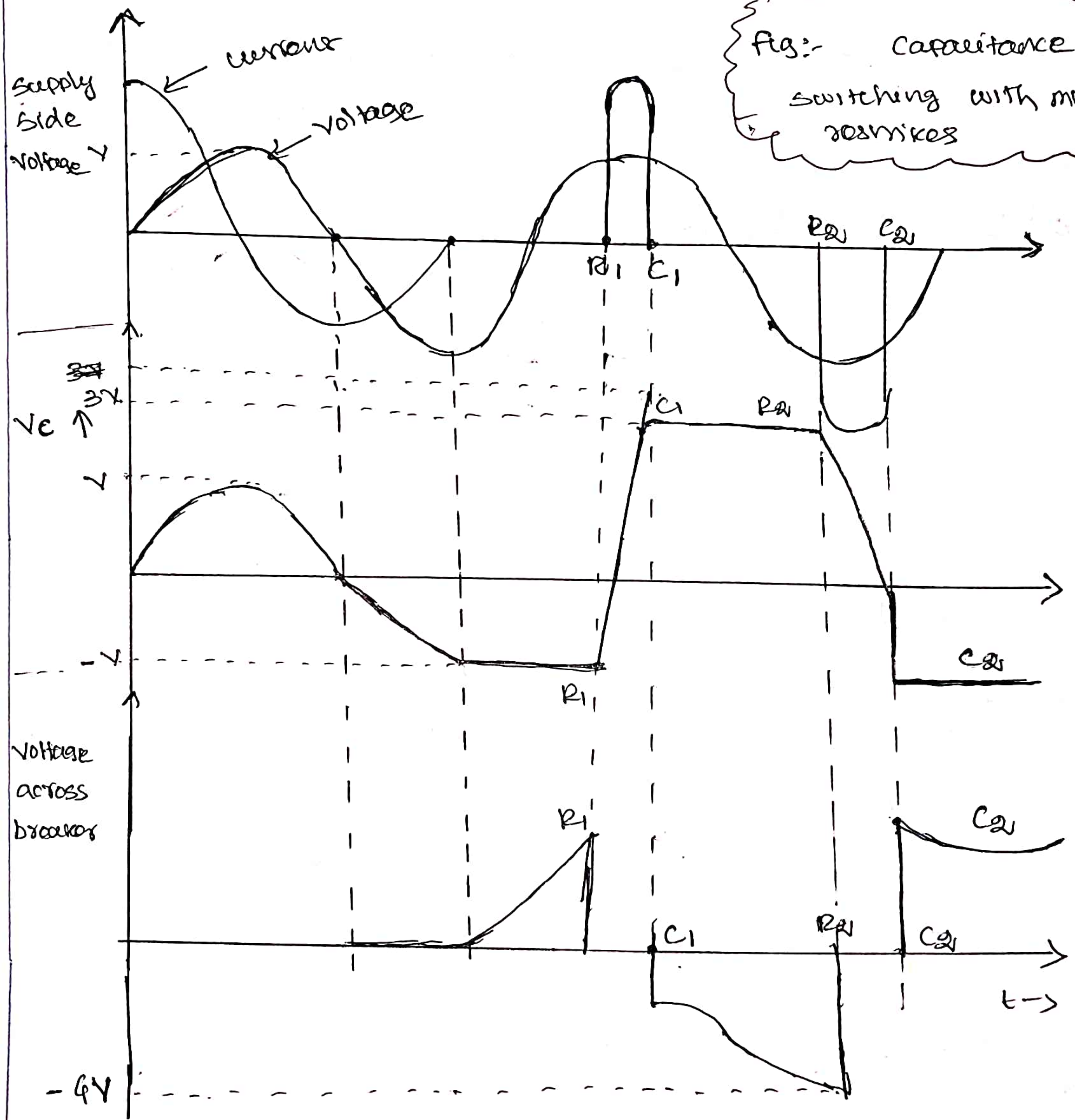


Fig:- Capacitance switching with multiple restrikes

* High voltage circuit breakers which have to perform capacitive current should be restrike free to avoid over voltages.

* In the above diagram $\boxed{R_a}$ - represents sequential restrikes and $\boxed{C_a}$ represents subsequent clearings

* ~~In~~ when multiple restriking occurs it is possible for a voltage of 4 p.u to be developed across the switch.

1.8 FERRO RESONANCE:-

"Ferro resonance or non linear resonance is a type of resonance in electric circuits which occurs when a circuit containing non linear inductance is fed from a source that has series capacitance and the circuit is subjected to a disturbance such as opening of switch.

* It can cause over voltages and over currents in an Electrical power system and can pose a risk to transmission and distribution equipment and to operational personnel.

Resonance:-

In linear resonance current and voltage are linearly dependent related in a manner that is frequency dependent.

Ferro resonance:-

* In the case of ferro resonance it is characterized by a sudden jump of voltage and current from one stable operating state to another one.

* The relationship between voltage and current is dependent not only on frequency but also on other factors

i.e (i) system voltage magnitude

(ii) initial flux condition

(iii) total loss

etc.,

* In the phenomenon of series resonance a very high voltage can appear across the elements of series L C Circuit when it is excited at or near its natural frequency.

Circuit diagram:-

* The situation where series resonance occur in transmission line & Transformer respectively are

(i) Compensation of Tr. line by series capacitance

(ii) Energising a transformer - single pole only closed.

(i) Series Capacitance:-

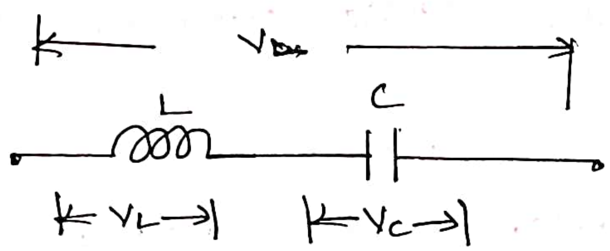
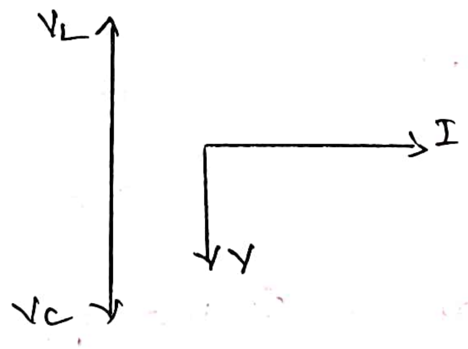


fig:- series compensation

* From the fig. The applied V is summation of voltage across the inductor and voltage across capacitor

ie $V = V_L + V_C$ - - - - - - - (1)

Phasor diagram:-



* From the phasor diagram, the voltage across the inductor leads the current in phase by 90° and the capacitor voltage lags the current by 90° (I - resonance).

* It is seen that both V_L and V_C can exceed V. ~~These~~ The voltage condition of this kind can be sustained and are therefore called as dynamic over voltages, rather than transients.

* ~~seen~~ such resonant conditions are to be avoided in power circuits. This phenomenon such that both V_L and V_C far exceed V is called as ferro resonance

Analysis:- (saturable Inductor)

* The voltage across the inductance will depend upon the frequency (ω) and current through a inductance function f(I) [Here f → function of I]

$$\boxed{\therefore\ V_L = \omega f(I)} \longrightarrow (2)$$

* V_L is plotted as a function of current in figure

* This voltage will lead the current by 90°

* The voltage across the capacitor is given by

$$\boxed{V_C = \frac{-I}{\omega C}} \quad \text{—— (3)}$$

* Minus sign indicating that it is antiphase with V_L and lags the current by 90°

* The total voltage will therefore be

$$\boxed{V = V_L + V_C} \quad \text{—— (4)}$$

$$\therefore \quad V = \omega f(I) - \frac{I}{\omega C}$$

(or)

$$V_L = V - V_C$$

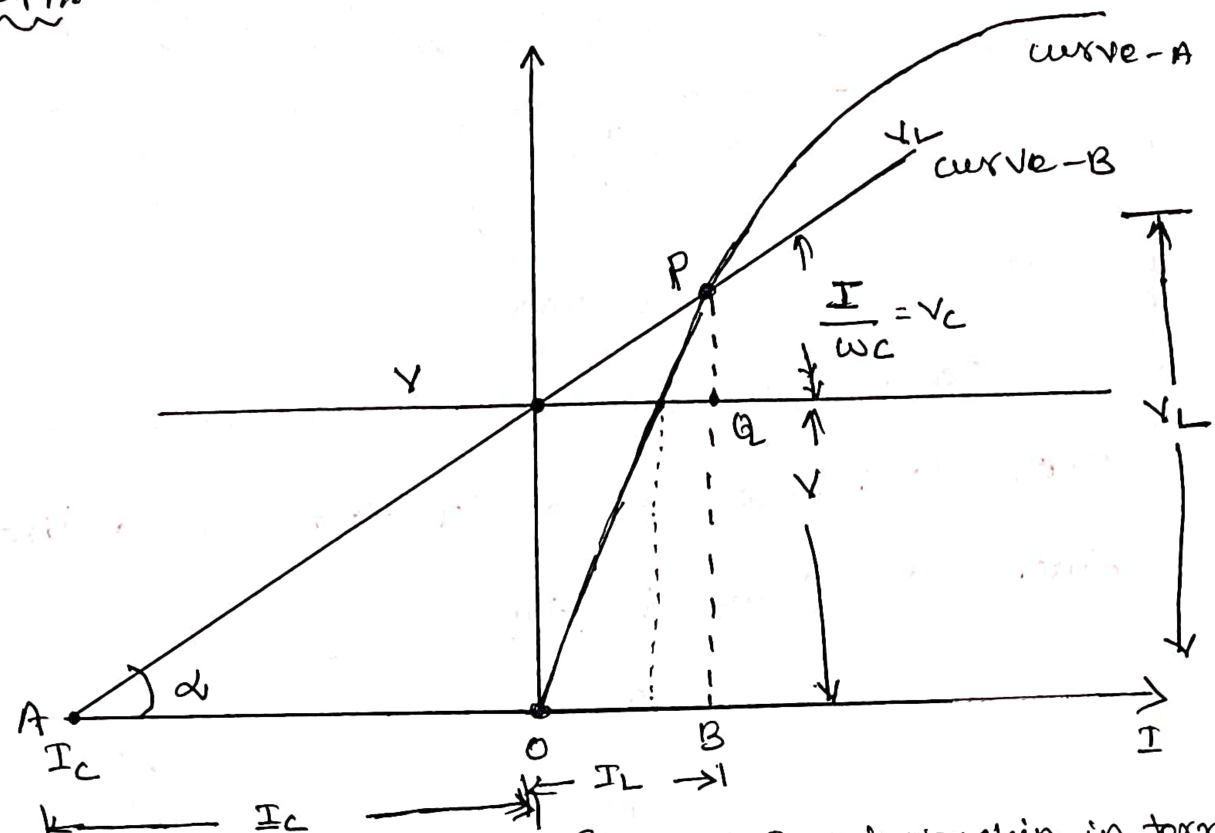
$$= V - \left(-\frac{I}{\omega C}\right)$$

$$\boxed{V_L = V + \frac{I}{\omega C}} \quad \text{—— (5)}$$

* Equation (5) represents that V_L has a fixed constituent 'V' and that is proportional to I

* This is represented and plotted in fig as the inclined straight line

Graphs



[Fig. V–I relationship in ferro resonant ckt

* Since both the curves represents (A&B) V_L the operating point must be where the two lines cross at 'P'

→ Point PQ → Capacitor voltage

PB → Inductor voltage $[V_L = V + \frac{I}{\omega C}]$

OB → current – I_L

OA → current – I_C

* If the voltage 'V' applied to the capacitor alone, it would take a much larger current 'I_C' but if applied to the inductor alone the current would be the smaller current 'I_L'

* The slope of Inclined line is given by

$$\boxed{\tan \alpha = \frac{1}{\omega C}}$$

Graph:- (Linear Inductor):-

* Ferro resonance in power circuit with series combination of a capacitor and a linear inductor is shown in fig.

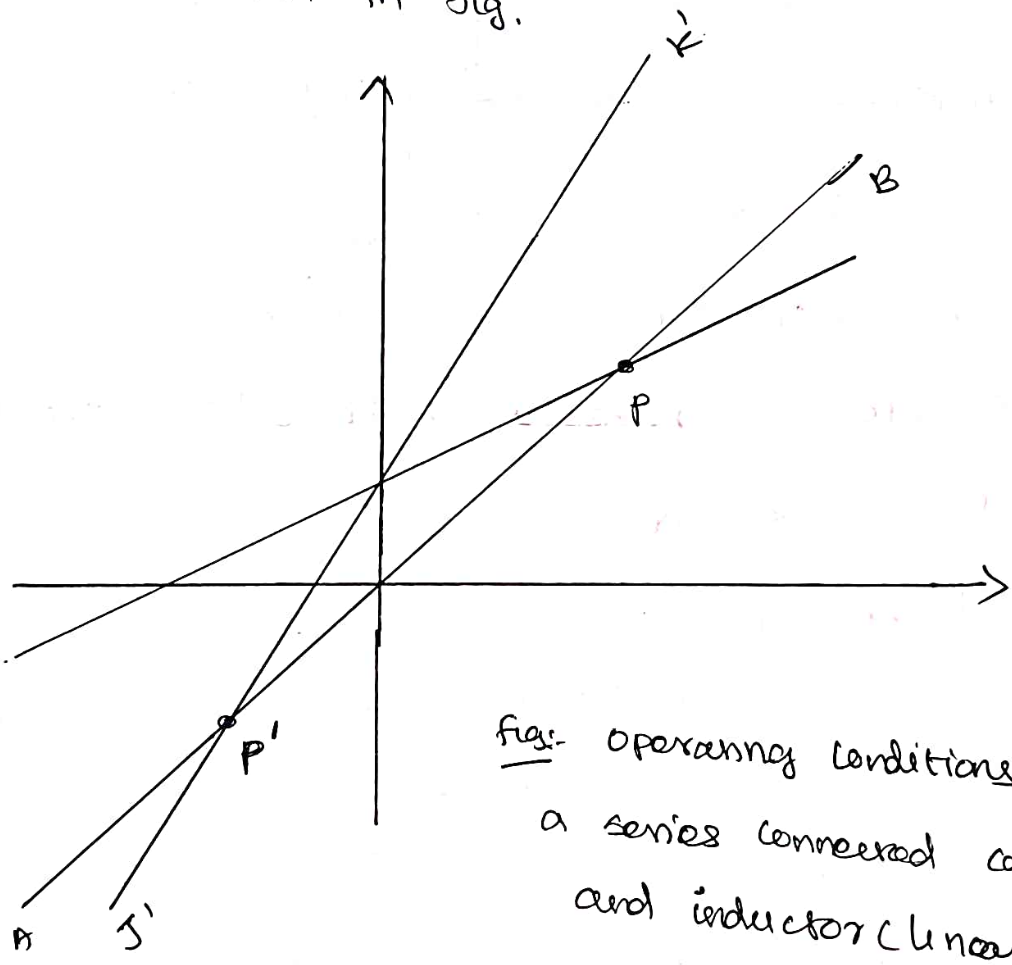


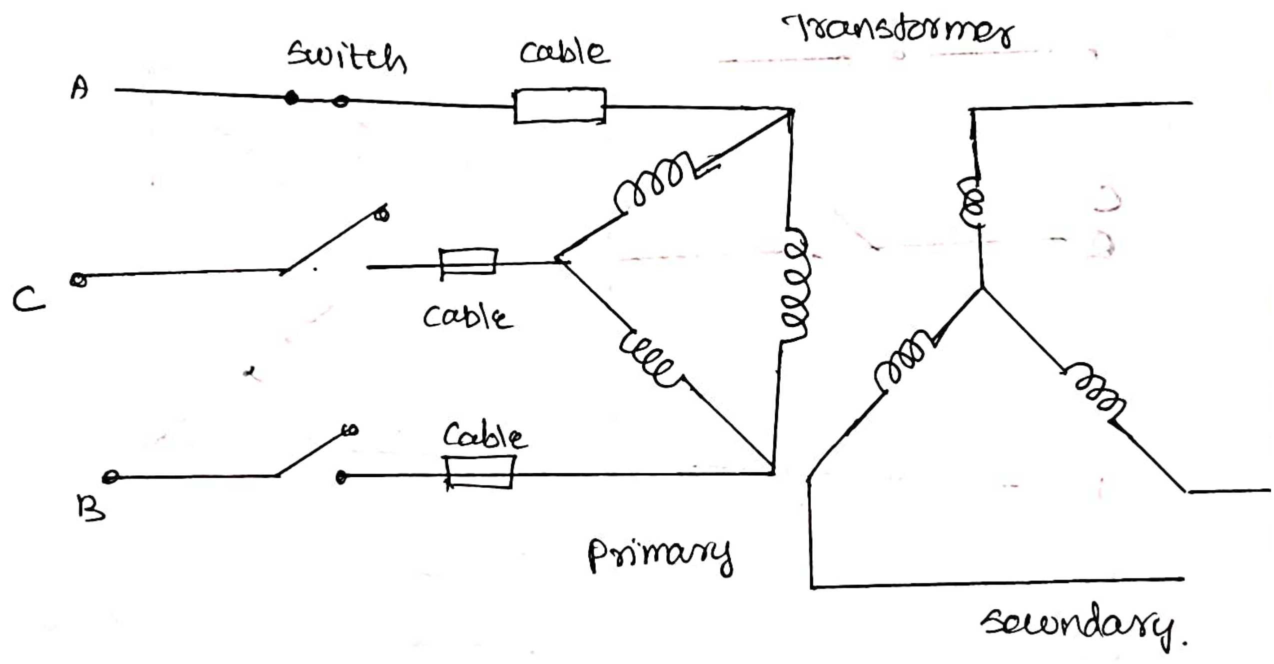
fig:- operating conditions with a series connected capacitor and inductor (linear)

* From the fig straight line A-B is the characteristics of the inductor

* The characteristics of the capacitor is given by JK or J'K' according to the value of capacitor, the operating point will be P or P'

* If the value of $\omega L > \frac{1}{\omega C}$ the operating point is at P, and if the value of $\omega L < \frac{1}{\omega C}$ the operating point is at P'

Ferro resonance – How it arises – Practical Example:-



* Fig. shows a switch used to energize and de-energize the primary of a transformer. The switch is interconnected to the primary by a length of cable.

* The switching device may be mounted at the top of a pole and a transformer on a nearby pad at ground level.

* Consider only one pole of switch is closed, then the transformer is not energised. Thus there is a path for the flow of current through two of the phase windings and the cable capacitance is shown in fig.

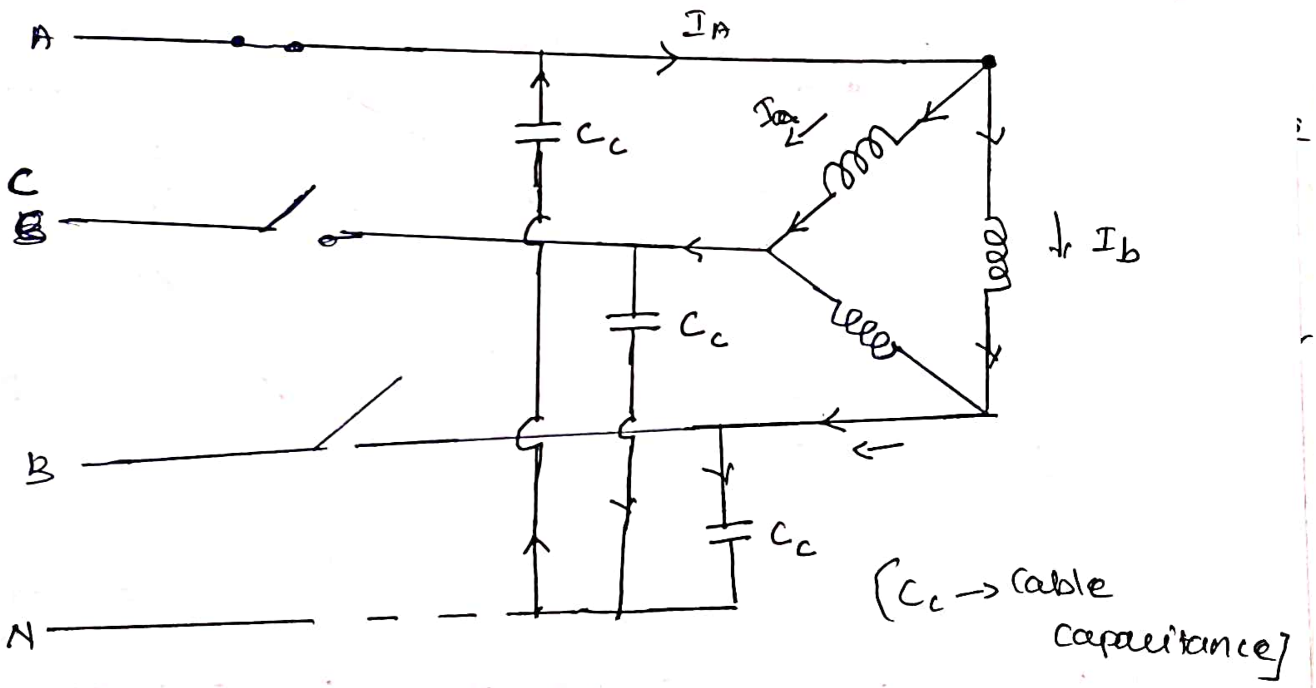


fig. Circumstance in which ferro resonance can occur.

* Dark line – flow of current path.

* This current flowing in a specified path can produce resonance and impress excessive voltage across the transformer and the cables on the un-energised phases.

* It can cause lightning arrestors connected at 'B' and 'C' bushings of the transformer to operate

* If the condition is sustained repeated operation can destroy the arrestors.

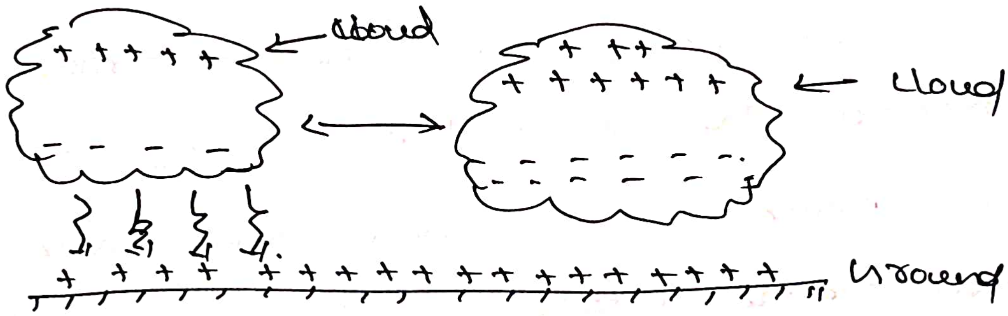
* There is possibility of ferro resonant overvoltage on Star-Delta ~~banks~~ transformer banks during single phase switchings as a function of transformer size and length of cable.

UNIT-III - LIGHTNING TRANSIENTS:-

3.1 Introduction - Lightning Transients:-

Definition:

"Lightning is a peak discharge in which charge accumulated in the clouds discharge into a neighbouring cloud or to the ground."



Types of Lightning strokes:-

There are two types of lightning strokes namely

1. Direct stroke
2. Indirect stroke.

1. Direct stroke:-

* In a direct stroke the lightning discharge is directly from the cloud to the subject equipment.
* From the line, the current path may be over the insulator down the pole to the ground.

2: Indirect stroke:-

Indirect stroke results from the electrostatically induced charges on the conductors due to the presence of charge clouds.

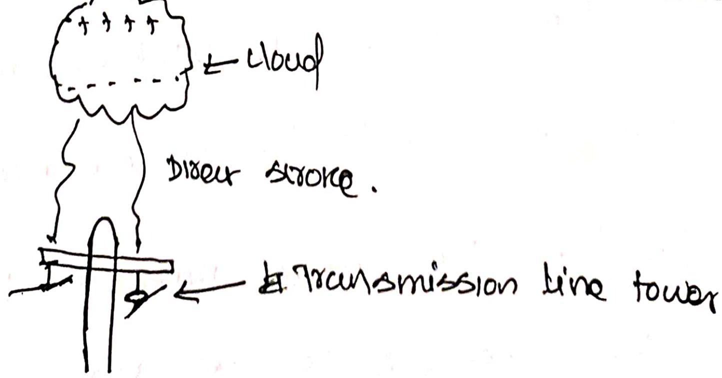
Effects of Direct and indirect stroke:-

1 Direct stroke Effects:-

In direct stroke directly strikes on structures, Transmission line, buildings (tall) etc.

* On because of this direct strike, generates the direct thermal effects and it leads to the melting of conductor, or creates the fire at the striking point.

* Protection against the direct effect of lightning is based on catching the current and discharging it to the earth. (lightning conductor, catcher rod etc



Indirect stroke Effects:-

- Lightning indirectly hit the electrical equipment.

over voltages due to lightning can reach the installation by ~~three~~ two means of access

(1) By conduction:-

Following direct strike on lines (Power, telecommunication, TV, etc.,) entering or exiting buildings by feedback from the earth via the earthing system, the protective conductors and the exposed conductive parts of equipment.

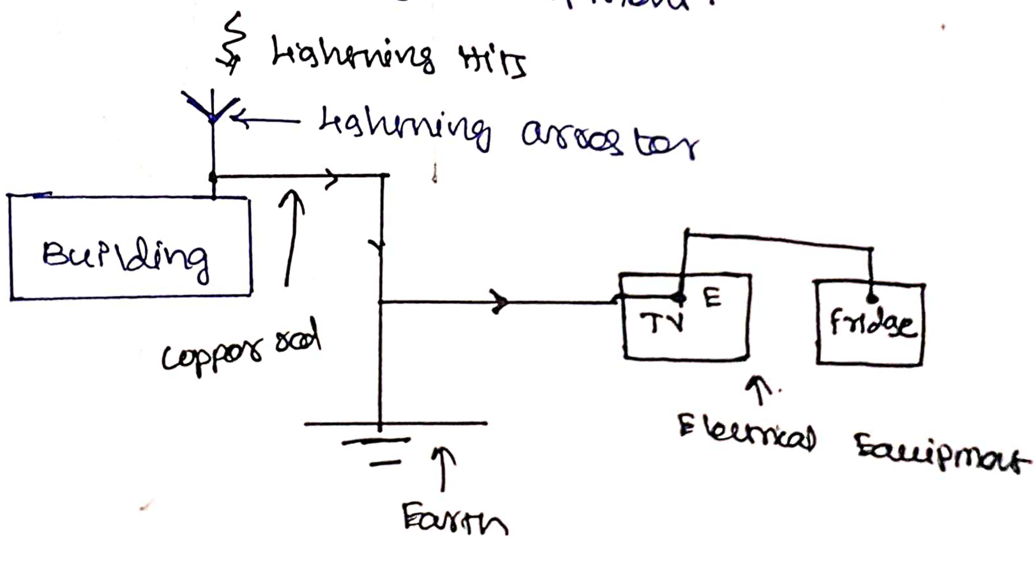


Fig. Effects due to conduction

FACTORS CONTRIBUTING TO GOOD LINE DESIGN :-

* Lightning and switching surges can occur occassionally in electric power transmission lines.
* These phenomena can produce high voltage level in a very short time that can damage insulation or can cause severe flashover.
* In order to ~~red~~ reduce the hazard that lightning poses to power system certain factors that determine the line performance must be understood.

Factors to be considered:-

a) The objective of good line design is to ~~red~~ reduce the no. of outages caused by lightning.

b) Try to keep the incidence of strokes to the system to a minimum.

c) Then try to minimize the effects of those strikes that do terminate on the system.

d) Lighening problems can be eliminated if all transmission was through tunnels at least 20 ft under the ground.

e) Tall towers are more vulnerable ~~to~~ than low goal post like structures. In order to prevent the lightning some adequate clearance must be provided.

f) High ground impedance or tower footing ~~re~~ resistence to be avoided.

g) High surge impedance in ground wires tower structures are to be avoided.

Transmission line over voltage protection:-

Over voltages due to lightning strokes can be avoided or minimized in practice by

(i) Shielding the overhead lines by using ground wires above the phase wires

(ii) Using Ground rods and counter-poise wires

(iii) Including ~~Protective devices including like~~ protective devices

(B)

(iii) Including protective devices like - Expulsion gaps protector tubes on the lines and surge divertors at the line terminations and substations.

(i) Lightning protections using shield wires or ground wires:-

* Ground wire is a conductor run parallel to the main conductor of the transmission line supported on the same tower and earthed at every equally and regularly spaced towers.

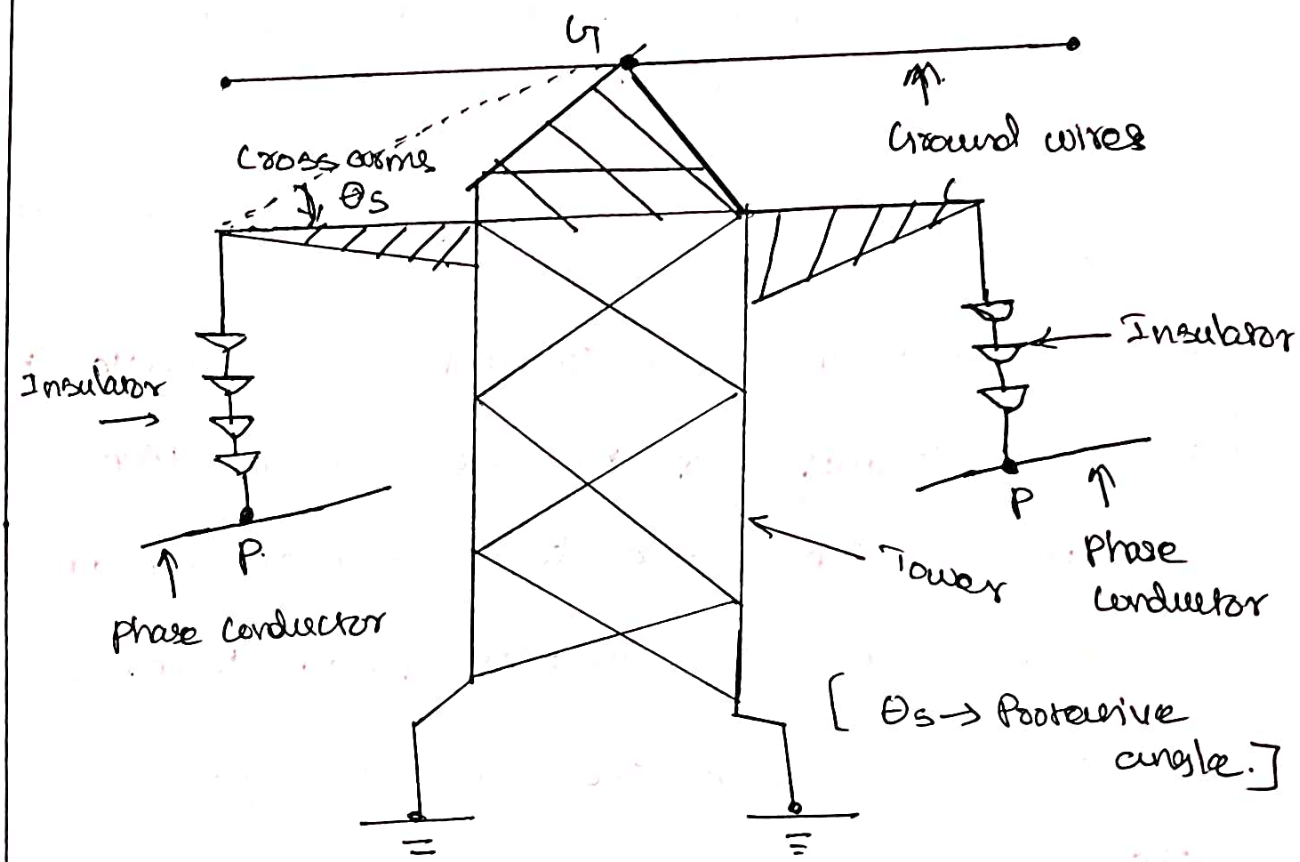
* It runs above the main conductor of the line.

* The ground wire shields the transmission line conductor from induced charges from clouds as well as from a lightning discharge.

* The arrangements of ground wires over the line conductor is shown in fig.

* There are two types of tower structures possible for supporting transmission line.

(i)



Line Protecting mechanism:-

* If a positively charged ~~gr~~ cloud is assumed to be above ~~a~~ the line it ~~may~~ includes the negative charge on the portion below it, of the transmission line.

* With the ground wire present both the ground wire and the line conductor get the induced charge.

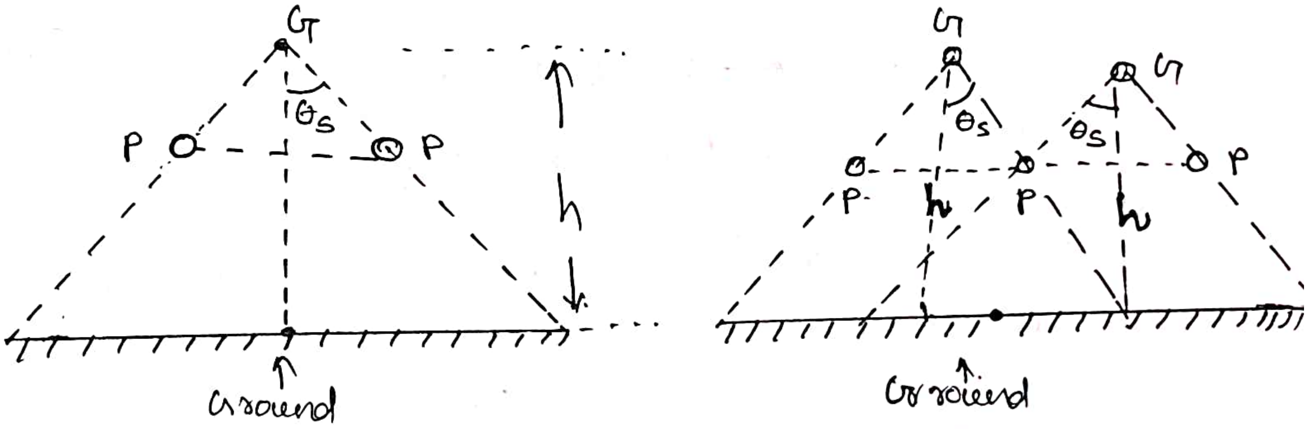
* But the ground wire is earthed at regular intervals and as such the induced charge is drained to the earth potential only.

* The shield wire may reduce the overvoltages associated with nearby strokes.

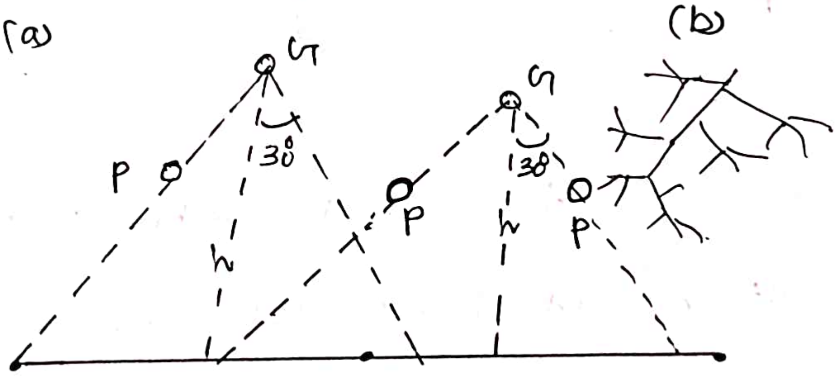
* The effective protection or shielding given by the ground wire depends on

(i) Height of the ground wire above the ground ch

(ii) Protection angle (or) shield angle θ_s (usually 30°)



(a) (b)



G - Ground wire P-P → phase wires θ_s → shielding angle h → height of the ground wire

* The shielding angle $\theta_s < 30^\circ$ was considered adequate for tower heights of 30m or less.

* The shielding wires may be one or more depending on the type of tower used.

* But for EHV lines the tower heights may be upto 50m, and the lightning strokes sometimes occur directly to the line wires as shown in fig. 'c'

The surge impedance of

(i) Transmission line < 500Ω (about 300Ω to 500Ω) ($Z_{Tr.L}$)

(ii) Ground wires → 100 to 150Ω (Z_g)

(iii) Towers — 10 to 50Ω (Z_T)

Order of surge impedance should be selected such that

$$\boxed{Z_T < Z_g < Z_{Tr.L}}$$

Uses:-

1. It is used for direct stroke protection of lines for voltages of 110kV and above.

2). To protect lines from attenuation of travelling waves setup in the line.

Fz01

Protection using COUNTER-POISE wires:-

- Counter poise wires are buried in ground at depth of 0.5 to 1m running parallel to the transmission line conductors and connected to the tower legs.
- a wire length may be 50 to 100m long. The arrangement of counter poise wire is shown in fig.

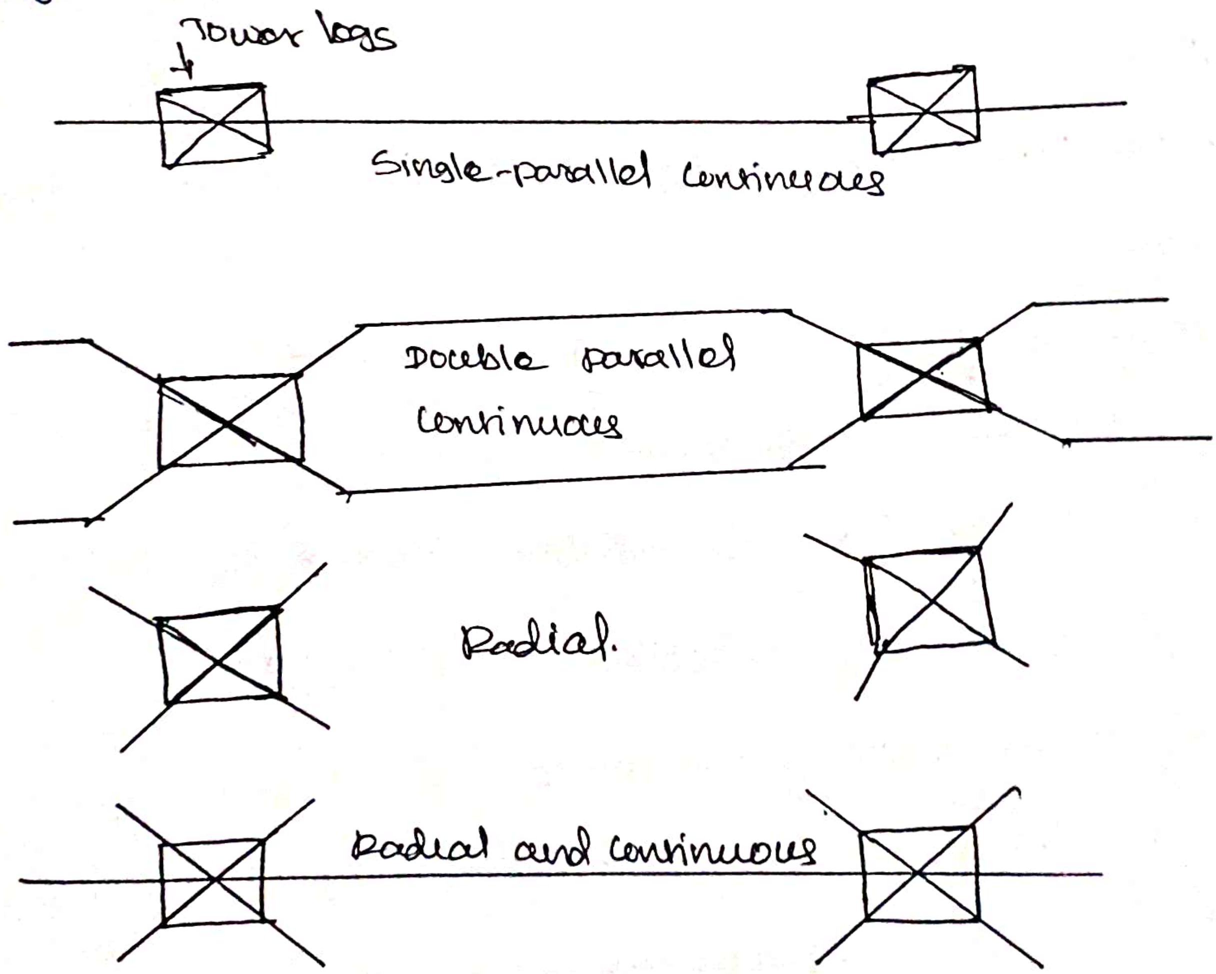


Fig: Arrangements of counter poise.

* When the lightning stroke incident on the tower, discharges first through the tower to the ground and discharges through the counter-poise.

For proper operation

- Leakage resistance of counter poise $<$ Surge Impedance.

* If lightning strikes a tower, current is injected and potential rises and flash-over of insulator disc takes place which results in a line to ground fault.

* So, the tower footing resistance value should be low.

Material used:- Galvanized steel wire.

TOWER FOOTING RESISTANCE:-

* It is the resistance offered by the tower footing to the dissipation of ground.

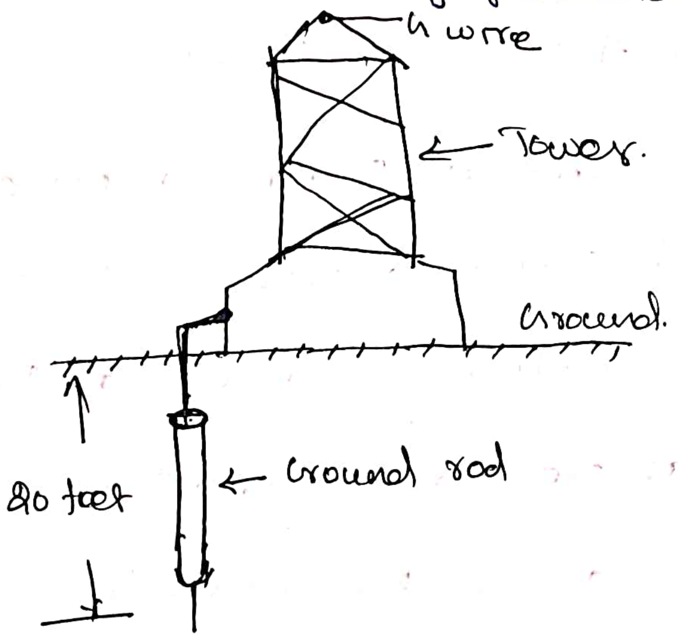
* Tower footing resistance is an important parameter that decides the effectiveness of a transmission line in an environment of dangerous atmospheric over voltages.

* High footing resistance causes back flashover on insulators
* The soil characteristic is one of the major effects that affect the value of tower footing resistance.
* Due to the various soil types and subsoil structure, different types of electrode arrangements are used as tower footing resistances
* When lightning strikes a tower, a travelling voltage is generated which travels back and forth along the tower, being reflected at the tower footing and at the tower top, thus raising the voltage across the cross arms and stressing the insulators.
* The insulator would flashover if this transient voltage exceeds its withstand level.
* The effective ground wire depends to a large extend on the tower footing resistance
* The tower top potential depends on this resistance.

Significance:-

* Low value of tower footing resistance results in less voltage stresses across line insulation.

* A tower footing resistance of 20 Ω for EHV lines and 10 Ω for HV lines provides sufficient lightning protection.



Transient Tower footing Resistance model:-

$$R_T = \frac{R_g}{\sqrt{1+\frac{I}{I_g}}}$$

Where $R_T \rightarrow$ Tower footing resistance in ohms

$R_g \rightarrow$ Tower footing Resistance at low current and low frequency in ohms.

$$\boxed{I_g = \frac{1}{2\pi} \frac{E_0 \rho_0}{R_g^2}}$$

$I_g \rightarrow$ Limiting current initiating soil ionization (kA)

$\rho_0 \rightarrow$ soil resistivity in ohm-meter

$E_0 \rightarrow$ soil ionization gradient (about 300kV/m)

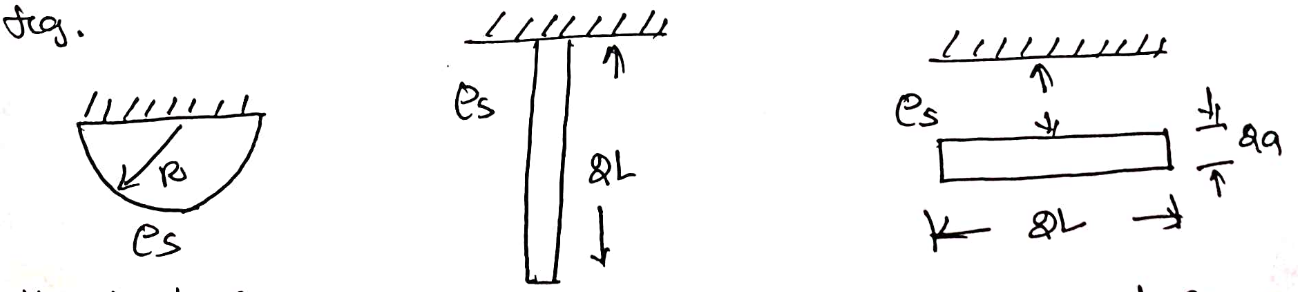
The most common type of tower footing resistance are

1) Hemisphere
2) Vertical driven rod
3) Buried horizontal wire (counter poise wires)

Soil resistivity has the following ranges

1. Sea water — 100 Ω-m
2. Moist soil — 101 Ω-m
3. Loose soil — 102 Ω-m
4. clay rock — 103 Ω-m

Electrodes shapes and their dimensions are shown in fig.



Hemisphere (a)

Vertical driven rod. (b)

Horizontal Buried wire counter poise (c)

Tower footing resistance depend on

(i) Type of configuration employed (Electrode)

(ii) soil resistivity

(iii) Electrode shapes:-

→ Used to reduce earth wire potential and stress on insulators at the time of stroke and also for safety.

→ Tower footing resistance will be 10Ω and should not be more than 20Ω under any condition throughout the year.

→ The soil resistivity - 100Ω-m

→ The Earth resistance depend upon soil resistivity (General 100Ω-m)

UNIT-IV

TRAVELLING WAVES ON TRANSMISSION LINES - COMPUTATION OF TRANSIENTS

"Travelling waves on transmission line is the voltage/current waves from the source end to the load end during the transient condition propagating as electromagnetic waves with a finite velocity. Hence it takes short time for load to receive the power."

* This gives rise to concept of travelling waves on transmission lines.

4.1 Computation of Transients in Transmission line:-

Two wire transmission line:-

Fig:4.1 Two wire Transmission line.

* Let us consider two wire lossless transmission line

* The transmission line consists of distributed line parameters 'R, L and C.

* These parameters are distributed throughout the length of transmission line.

* Let L and C be the inductance and capacitance per unit length of the line.

Equivalent circuit:-

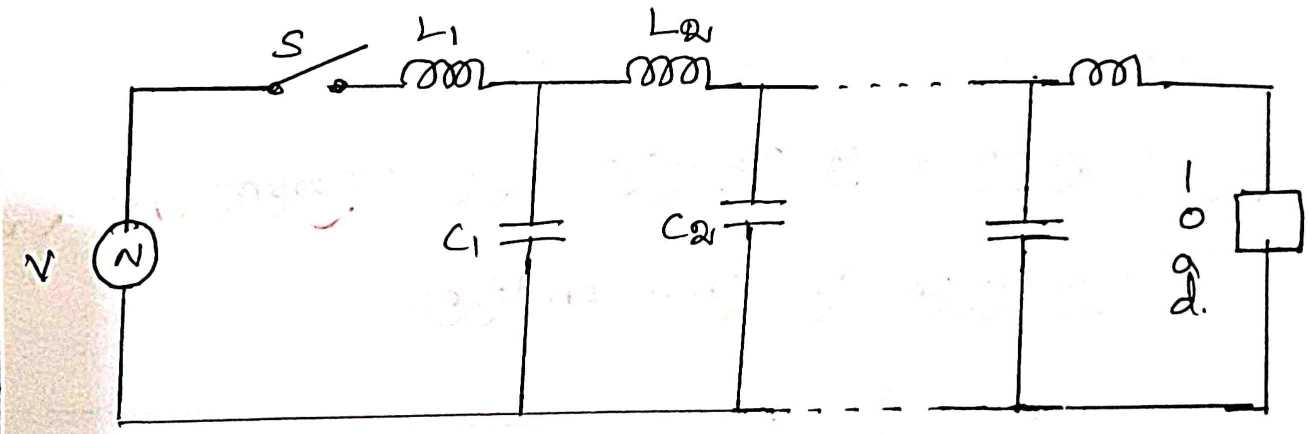


fig: 4.2 Equivalent circuit of Tr. line.

* When the switch is closed at the transmission line's starting end, voltage will not appear instantaneously at the other end.

* This is caused by transient behaviour of Inductor and capacitor that are present in Tr. line.

* Therefore when the switch is closed the voltage will build up gradually over the line conductors.

* when the switch 'S' is closed current flows through the first inductance to charge the first capacitance 'C_1'.

* After charging of C_1 there is a voltage across the second section of the line and current commences to flows through second 'L_2' to charge the second capacitor 'C_2'

* This process is followed throughout the length of the line.

Computation of Transients:-

* Let it be assumed that after a time 'Δt' a length 'Δx' of line has been so charged. If the capacitance of the line is 'C' ~~per~~ farads per meter a charge Q will have been imparted to the line

Hence the stored charge in shunt capacitance

$$\boxed{Q = CV\Delta x} \text{ - - - - - - } (1)$$

W.K.T

$$I = \frac{dQ}{dt} \text{ ——— } (2)$$

subs (1) in (2)

$$\therefore I = \frac{d}{dt} \cdot (CV \cdot \Delta x)$$

Diff eqn (1) w.r.to 't'

$$\frac{dQ}{dt} = C \cdot V \cdot \frac{\Delta x}{\Delta t}$$

$$\boxed{\frac{dQ}{dt} = C \cdot V \cdot \frac{dx}{dt}} \text{ ——— } (3)$$

subs (3) in (2)

$$I = C \cdot V \cdot \frac{dx}{dt}$$

[$\frac{dx}{dt}$ = velocity of travelling wave

$\frac{dx}{dt} = p$]

$$\boxed{\therefore I = C \cdot V \cdot p} \text{ ——— } (4)$$

* The flux linkages can be defined in terms of the line inductance [for unit distance]

$$\boxed{q = LI \cdot \Delta x} \text{ ——— } (5)$$

* The induced emf is the rate of change of flux linkage is being induced in the loop formed by conductors and wave front.

* The induced emf can be expressed as

$$\boxed{V = \frac{d\phi}{dt}} \quad \text{—— (6)}$$

* Diff equation no. 5 w.r.to 't' and substitute in eqn no. (6)

$$\phi = L \cdot I \, \Delta x$$

$$\frac{d\phi}{dt} = L \cdot I \, \frac{\Delta x}{\Delta t}$$

$$\frac{d\phi}{dt} = L \cdot C \cdot V \cdot P \cdot \frac{\Delta x}{\Delta t}$$

$$\boxed{\frac{d\phi}{dt} = LCVP^2} \quad \text{—— (7)} \quad \left[\because \frac{\Delta x}{\Delta t} = P\right]$$

sub equation no. (7) in (6)

$$\cancel{V} = LC\cancel{V}P^2$$

P → Velocity of propagation

$$P^2 = \frac{1}{LC}$$

$$\boxed{P = \frac{1}{\sqrt{LC}}} \quad \text{—— (8)}$$

W.K.T

$$\boxed{L = \frac{\mu_0}{\pi} \ln \frac{d}{r} \ H/m} \quad \text{--- (9)}$$

$$\boxed{C = \frac{\pi \varepsilon_0}{\ln d/r} \ F/m} \quad \text{--- (10)}$$

sub equation no. (9)&(10) in equation no. 8

$$p = \frac{1}{\sqrt{\frac{\mu_0}{\cancel{\pi}} \cancel{\ln(d/r)} \times \frac{\cancel{\pi} \varepsilon_0}{\cancel{\ln(d/r)}}}}$$

$$\boxed{\therefore \ p = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}} \quad \text{--- (11)}$$

Subs (18) in (4)

$$\therefore I = C \cdot V \cdot P$$

$$I = C \cdot V \cdot \frac{1}{\sqrt{LC}}$$

$$\therefore \frac{V}{I} = \frac{\sqrt{LC}}{C} \qquad [C = \sqrt{C} \cdot \sqrt{C}]$$

$$= \frac{\sqrt{L} \cdot \cancel{\sqrt{C}}}{\sqrt{C} \cdot \cancel{\sqrt{C}}}$$

$$\frac{V}{I} = \sqrt{\frac{L}{C}}$$

$$\boxed{\therefore Z_0 = \sqrt{\frac{L}{C}}}$$

* The above expression is the ratio of voltage and current having the dimensions of Impedance.

∴ It is called surge Impedance.

* So, Surge Impedance is the square root of ratio of series inductance 'L' per unit length of line and shunt capacitance 'C' per unit length of line.

* This value will not change due to change in length of line.

* The surge impedance for a typical transmission line is about 400 ohm and for a cable is around 40 ohm

* A travelling wave of voltage passes along the line at a velocity approaching the speed of light establishing an electric field between the conductors.

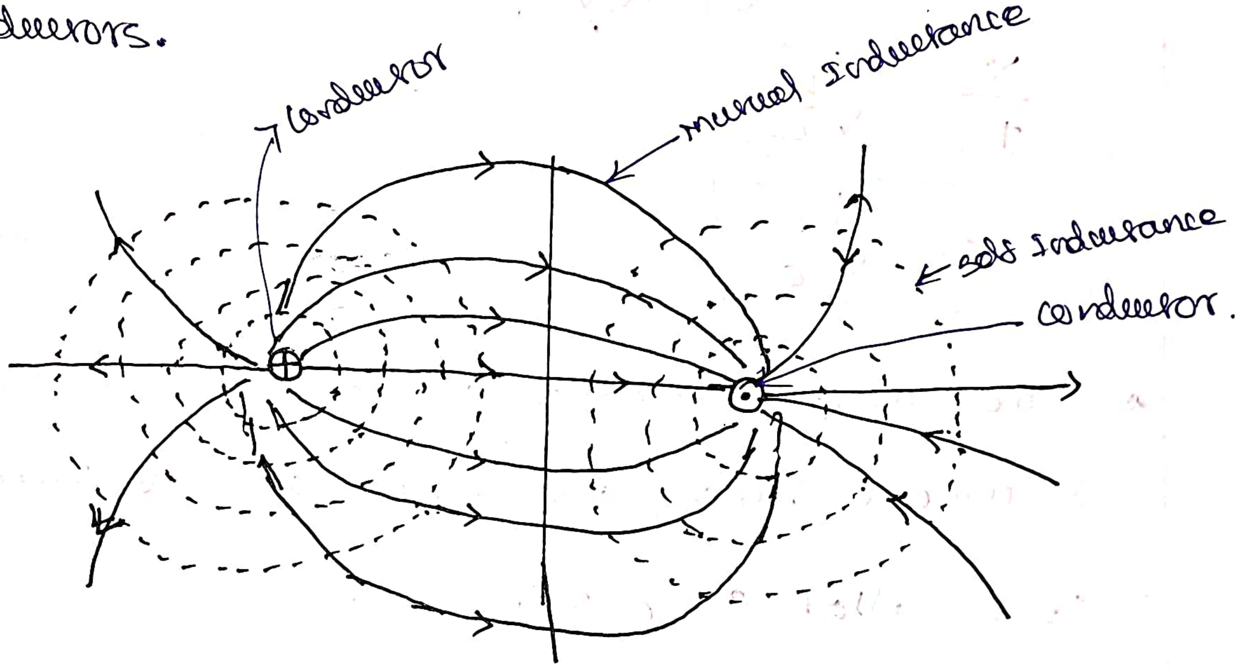


Fig:- Electro magnetic field of two conductor transmission line

* The voltage wave is accompanied by a current wave of amplitude $\frac{V}{Z_0}$ which in turn creates a magnetic field in the surrounding space.

TRANSIENT RESPONSE OF SYSTEM WITH SERIES AND SHUNT LUMPED PARAMETERS AND DISTRIBUTED LINES

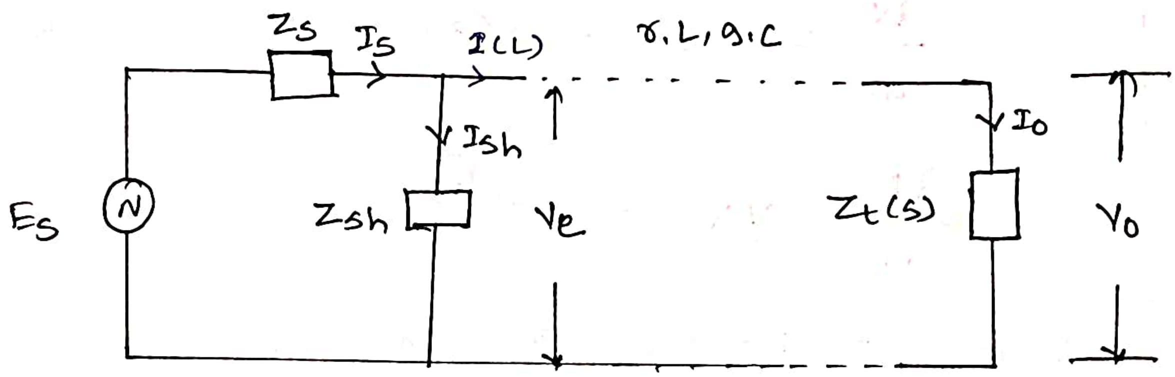


Fig:- Equivalent circuit of transmission line

Let

$E_s \rightarrow$ Source voltage

$Z_s \rightarrow$ series impedance of transmission line

$Z_{sh} \rightarrow$ Shunt Impedance of shunt compensating reactor

$Z_t \rightarrow$ Impedance of Transformer,

$V_e \rightarrow$ Voltage near to source End

$I_o \rightarrow$ Load current

$V_o \rightarrow$ Voltage at far End of transmission line.

Let

$$Z = R + j\omega L = R + sL \quad \text{—(1)}$$

$$Y = G + sC = G + sC \quad \text{—(2)}$$

At any point x from the termination point which has impedance 'Zt', the steady state equation of circuit are

$$\frac{\partial V}{\partial x} = Z I \quad \text{——} (3)$$

$$\frac{\partial I}{\partial x} = Y V \quad \text{——} (4)$$

Solution of Equation (3) & (4) are

$$V = Ae^{px} + Be^{-px} \quad \text{——} (5)$$

for current

$$(3) \Rightarrow \quad I = \frac{1}{Z} \cdot \frac{\partial V}{\partial x} \quad \text{——} (6)$$

Diff equation no. (5) w.r.to 'x' and substitute in (6)

$$I \text{ ie } \frac{\partial V}{\partial x} = p \cdot Ae^{px} - Be^{-px} \cdot p$$

$$\therefore \quad I = \frac{1}{Z}\left[pAe^{px} - pBe^{-px}\right]$$

$$\boxed{I = \frac{p}{Z}\left[Ae^{px} - Be^{-px}\right]} \quad \text{——} (7)$$

To determine constant A & B:-

consider the following boundary condition

(i) At $x = L$, $V = V_e$ . . . . (8)

(ii) At $x = 0$, $V(0) = I(0) Z_t$ (or) $I(0) = \frac{V(0)}{Z_t}$ . . . . (9)

Apply Equation number (8) & (9) in equation no: 5

(5) $\Rightarrow$

$$V = Ae^{px} + Be^{-px}$$

$$\boxed{\therefore V_e = Ae^{PL} + Be^{-PL}} \quad \text{------ (10)}$$

Similarly put $x = 0$ & $V = V(0)$

$$\therefore V(0) = Ae^{P\times 0} + Be^{-P\times 0}$$

$$\boxed{\therefore V(0) = A + B} \quad \text{------ (11)}$$

Now Apply second boundary condition in equation No:-(7)

(7) $\Rightarrow$ $I = \frac{P}{Z}\left[Ae^{px} - Be^{-px}\right]$ [put $x = 0$]

$$I(0) = \frac{P}{Z}\left[Ae^{P\times 0} - Be^{-P\times 0}\right]$$

$$I(0) = \frac{P}{Z}\left[A - B\right]$$

$$\boxed{\frac{V(0)}{Z_t} = \frac{P}{Z}\left[A - B\right]} \quad \text{------ (12)}$$

W.K.T $Z_0 = \sqrt{\frac{L}{C}}$

We may also write

$$Z_0 = \frac{Z}{P}$$

(or) $$\boxed{\frac{P}{Z} = \frac{1}{Z_0}} \quad -(13)$$

sub (13) in (12)

$$(12) \Rightarrow \frac{V(0)}{Z_t} = \frac{1}{Z_0}[A-B]$$

$$\boxed{V(0) = \frac{Z_t}{Z_0}[A-B]} \quad -(14)$$

Equating Equation no. (11) & (14)

$$A+B = \frac{Z_t}{Z_0}[A-B]$$

$$Z_0[A+B] = Z_t[A-B]$$

$$Z_0A + Z_0B = Z_tA - Z_tB$$

$$Z_0A - Z_tA = -Z_0B - Z_tB$$

$$A(Z_0 - Z_t) = -B[Z_0 + Z_t]$$

$$\boxed{A(Z_0 - Z_t) + B(Z_0 + Z_t) = 0} \quad -(15)$$

Note:- For Ref:-

$$P = \sqrt{ZY}$$

$$= \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$\therefore Z_0 = \frac{R+j\omega L}{\sqrt{R+j\omega L}\cdot\sqrt{G+j\omega C}}$$

$$= \frac{\sqrt{R+j\omega L}\cdot\sqrt{R+j\omega L}}{\sqrt{R+j\omega L}\cdot\sqrt{G+j\omega C}}$$

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

If $R=0$; $G=\infty$

$$\therefore Z_0 = \sqrt{\frac{L}{C}}.$$

To find A & B

(15) $\Rightarrow$ $B(Z_0 + Z_t) = -A(Z_0 - Z_t)$

$$\boxed{B = \frac{-A(Z_0 - Z_t)}{Z_0 + Z_t}} \quad \text{—(16)}$$

sub (16) in (10)

(10) $\Rightarrow$ $Ae^{PL} + Be^{-PL} = V_e$

$$Ae^{PL} - A\left[\frac{Z_0 - Z_t}{Z_0 + Z_t}\right] e^{-PL} = V_e$$

$$Ae^{PL}[Z_0 + Z_t] - A[Z_0 - Z_t]\, e^{-PL} = V_e(Z_0 + Z_t)$$

$$A\left[e^{PL}[Z_0 + Z_t] - e^{-PL}(Z_0 - Z_t)\right] = V_e[Z_0 + Z_t]$$

$$\therefore A = \frac{V_e[Z_0 + Z_t]}{(Z_0 + Z_t)e^{PL} - [Z_0 - Z_t]e^{-PL}} \quad \text{—(17)}$$

$$\boxed{\therefore A = \frac{V_e[Z_0 + Z_t]}{(Z_0 + Z_t)e^{PL} + (Z_t - Z_0)e^{-PL}}} \quad \text{—(18)}$$

sub (18) in (16)

$$\therefore B = \frac{V_e \cancel{(Z_0+Z_t)} (Z_t - Z_0)}{\left((Z_0+Z_t)e^{PL} + (Z_t-Z_0)e^{-PL}\right)\cancel{(Z_0+Z_t)}}$$

$$\boxed{\therefore \quad B = \frac{V_e (Z_t - Z_0)}{(Z_0+Z_t)e^{PL} + (Z_t-Z_0)e^{-PL}}} \quad -(19)$$

sub (18) & (19) in equation no: (5)

(5) $\Rightarrow$ (5) $\Rightarrow$ $V = Ae^{Px} + Be^{-Px}$

$$\therefore V = \frac{(Z_t+Z_0) V_e e^{Px}}{(Z_0+Z_t)e^{PL} + (Z_t-Z_0)e^{-PL}} + \frac{V_e (Z_t - Z_0) e^{-Px}}{(Z_t+Z_0)e^{PL} + (Z_t-Z_0)e^{-PL}}$$

$$\therefore V = \frac{V_e \left[(Z_t+Z_0)e^{Px} + (Z_t-Z_0)e^{-Px}\right]}{(Z_0+Z_t)e^{PL} + (Z_t-Z_0)e^{-PL}}$$

$$\therefore V = \frac{V_e \left[Z_t e^{Px} + Z_0 e^{Px} + Z_t e^{-Px} - Z_0 e^{-Px}\right]}{Z_0 e^{PL} + Z_t e^{PL} + Z_t e^{-PL} - Z_0 e^{-PL}}$$

$$\therefore V = \frac{V_e\left[(e^{px}+e^{-px})Z_t + Z_o(e^{px}-e^{-px})\right]}{Z_t(e^{pL}+e^{-pL}) + Z_o\left[e^{pL}-e^{-pL}\right]}$$

$$\therefore V = \frac{V_e \cdot \cancel{Z_t}\left(\left[e^{px}+e^{-px}\right] + \frac{Z_o}{Z_t}\left[e^{px}-e^{-px}\right]\right)}{\cancel{Z_t}\left[e^{pL}+e^{-pL}\right] + \frac{Z_o}{Z_t}\left[e^{pL}-e^{-pL}\right]}$$

$$\boxed{\therefore V = \frac{\left[(e^{px}+e^{-px}) + \frac{Z_o}{Z_t}\left[e^{px}-e^{-px}\right]\right]V_e}{(e^{pL}+e^{-pL}) + \frac{Z_o}{Z_t}\left[e^{pL}-e^{-pL}\right]}} \tag{20}$$

W.K.T

$$\cosh x = \frac{e^{x}+e^{-x}}{2}; \quad \sinh x = \frac{e^{x}-e^{-x}}{2} \tag{21}$$

$$\therefore V = \left[\frac{2\cosh px + (2\sinh px)\frac{Z_o}{Z_t}}{2\cosh pL + \frac{Z_o}{Z_t}2\sinh pL}\right]V_e$$

$$\boxed{\therefore V = V_e\left[\frac{\cosh px + \frac{Z_o}{Z_t}\sinh px}{\cosh pL + \frac{Z_o}{Z_t}\sinh pL}\right]} \tag{22}$$

similarly for current 'I'

$$⑨ \Rightarrow \quad I = \frac{P}{Z}\left[Ae^{Px} - Be^{-Px}\right]$$

sub(18) & (19) in equation no:-(7)

$$I = \frac{P}{Z}\left[\frac{Ve\,(Zo+Zt)e^{Px}}{(Zo+Zt)e^{PL}+(Zt-Zo)e^{-PL}} - \frac{Ve\,(Zt-Zo)\,e^{-Px}}{(Zt+Zo)e^{PL}+(Zt-Zo)e^{-PL}}\right]$$

$$I = \frac{P}{Z}\left[\frac{Ve\left\{\left[Zo\,e^{Px}+Zt\,e^{Px}\right] - \left[Zt\,e^{-Px} - Zo\,e^{-Px}\right]\right\}}{Zo\,e^{PL}+Zt\,e^{PL}+Zt\,e^{-PL}-Zo\,e^{-PL}}\right]$$

$$I = \frac{P}{Z}\left[\frac{Ve\left[Zt\,(e^{Px}-e^{-Px}) + Zo\,(e^{Px}+e^{-Px})\right]}{Zt\left[e^{PL}+e^{-PL}\right] + Zo\left[e^{PL}-e^{-PL}\right]}\right]$$

$$I = \frac{P}{Z}\left[\frac{Ve\cdot \cancel{Zt}\left[e^{Px}-e^{-Px}\right] + \frac{Zo}{Zt}\left[e^{Px}+e^{-Px}\right]}{\cancel{Zt}\left[\left[e^{PL}+e^{-PL}\right] + \frac{Zo}{Zt}\left(e^{PL}-e^{-PL}\right)\right]}\right]$$

$$I = \frac{P}{Z}\left[\frac{Ve\;\; 2\sinh Px + \frac{Zo}{Zt}\,2\cosh Px}{2\cosh PL + 2\,\frac{Zo}{Zt}\sinh PL}\right]$$

$$I = \frac{P}{Z}\left[\frac{\sinh px + \frac{Z_0}{Z_t}\cosh px}{\cosh pL + \frac{Z_0}{Z_t}\sinh pL}\right] V_e \qquad \left[\because Z_0 = \frac{Z}{P},\ \frac{1}{Z_0} = \frac{P}{Z}\right]$$

$$\boxed{I = \frac{1}{Z_0}\left[\frac{\sinh px + \frac{Z_0}{Z_t}\cosh px}{\cosh pL + \frac{Z_0}{Z_t}\sinh pL}\right] V_e} \quad \text{—— (23)}$$

Current and Voltage at both the ends of the Transmission line:-

(i) At x=0; ie at the end of the line:-

sub equation no:- 22

$$V(0) = \left[\frac{\cosh p\times 0 + \frac{Z_0}{Z_t}\sinh p\times 0}{\cosh pL + \frac{Z_0}{Z_t}\sinh pL}\right] V_e \qquad [\because \cos 0 = 1]$$

$$\boxed{\therefore V(0) = \frac{V_e}{\cosh pL + \frac{Z_0}{Z_t}\sinh pL}} \quad \text{—— (24)}$$

similarly for current

(23) $\Rightarrow$ $$I(0) = \frac{1}{Z_0}\left[\frac{\sinh Px0 + \frac{Z_0}{Z_t}\cosh x0 \times P}{\cosh PL + \frac{Z_0}{Z_t}\sinh PL}\right] V_e$$

$$I(0) = \frac{1}{Z_0}\left[\frac{Z_0/Z_t}{\cosh PL + \frac{Z_0}{Z_t}\sinh PL}\right] V_e$$

$$I(0) = \frac{V_e}{Z_t\left[\cosh PL + \frac{Z_0}{Z_t}\sinh PL\right]}$$

$$\therefore \boxed{I(0) = \frac{V(0)}{Z_t}} \quad \text{——} (25) \qquad \left[\because (24) \Rightarrow V(0) = \frac{V_e}{\cosh PL + \frac{Z_0}{Z_t}\sinh PL}\right]$$

(ii) At $x = L$ ie at the entrance of the line :-

subt $\boxed{x = L}$ in equation no:- (22) & (23)

$$V(L) = \left[\frac{\cosh PL + \frac{Z_0}{Z_t}\sinh PL}{\cosh PL + \frac{Z_0}{Z_t}\sinh PL}\right] V_e$$

$$\therefore \boxed{V(L) = V_e} \quad \text{——} (26)$$

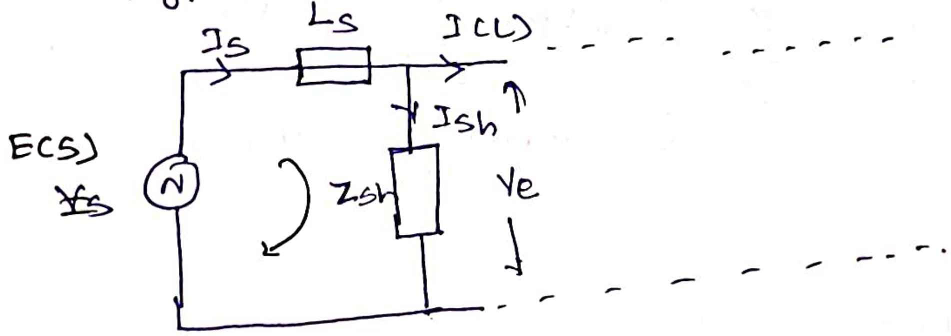
similarly for current

$$I(L) = \cancel{\frac{1}{Z_0}} \left[\frac{\cancel{\sinh pL + \frac{Z_0}{Z_t} \sinh pL}}{\cosh pL + \frac{Z_0}{Z_t} \sinh} \right.$$

$$\boxed{I(L) = \frac{1}{Z_0} \left[\frac{\sinh pL + \frac{Z_0}{Z_t} \cosh pL}{\cosh pL + \frac{Z_0}{Z_t} \sinh pL} \right] V_e} \quad \text{--- (87)}$$

Calculation of lumped parameter:-

From fig.



$$\boxed{I_{sh} = \frac{V_e}{I_{sh}}} \quad \text{--- (88)} \qquad \boxed{I_s = I_{sh} + I(L)} \quad \text{--- (89)}$$

Apply KVL

$$E(S) = V_s + V_{sh}$$

$$E(S) = Z_s I_s + V_e$$

$$V_e = E(S) - Z_s I_s \leftarrow \text{(sub (89) for } I_s)$$

$$V_e = E(S) - Z_s [I_{sh} + I(L)]$$

$$\boxed{V_e = E(s) - Z_s I_{sh} + Z_s I(L)} \quad \text{--- (30)}$$

{sub (29) & (27) in 30}

$$V_e = E(s) - \frac{V_e}{Z_{sh}} Z_s - \frac{Z_s}{Z_0}\left[\frac{\sinh PL + \frac{Z_0}{Z_t}\cosh PL}{\cosh PL + \frac{Z_0}{Z_t}\sinh PL}\right] V_e$$

$$E(s) = V_e + \frac{V_e}{Z_{sh}} Z_s + \frac{Z_s}{Z_0}\left[\frac{\sinh PL + \frac{Z_0}{Z_t}\cosh PL}{\cosh PL + \frac{Z_0}{Z_t}\sin hPL}\right] V_e$$

$$V_e\left[1 + \frac{Z_s}{Z_{sh}} + \frac{Z_s}{Z_0}\left[\frac{\sinh PL + \frac{Z_0}{Z_t}\cosh PL}{\cosh PL + \frac{Z_0}{Z_t}\sinh PL}\right]\right] = E(s)$$

$$V_e\left[\cosh PL + \frac{Z_0}{Z_t}\sinh PL + \frac{Z_s}{Z_{sh}}\cosh PL + \left(\frac{Z_s}{Z_{sh}} \times \frac{Z_0}{Z_t}\right)\sinh PL + \frac{Z_s}{Z_0}\sinh PL + \frac{Z_s}{\cancel{Z_0}} \times \frac{\cancel{Z_0}}{Z_t}\cosh PL\right]$$

$$= E(s)\left[\cosh PL + \frac{Z_0}{Z_t}\sinh PL\right]$$

$$V_e\left[\left(1 + \frac{Z_s}{Z_{sh}} + \frac{Z_s}{Z_t}\right)\cosh PL + \left(\frac{Z_0}{Z_t} + \frac{Z_s Z_0}{Z_{sh} Z_t} + \frac{Z_s}{Z_0}\right)\sinh PL\right]$$

$$= E(s)\left[\cosh PL + \frac{Z_0}{Z_t}\sinh PL\right]$$

$$\therefore V_e = \frac{E(s)\left[\cosh PL + \frac{Z_0}{Z_t}\sinh PL\right]}{\left[1+\frac{Z_s}{Z_{sh}}+\frac{Z_s}{Z_t}\right]\cosh PL + \left(\frac{Z_0}{Z_t}+\frac{Z_s Z_0}{Z_{sh} Z_t}+\frac{Z_s}{Z_0}\right)\sinh PL} \quad \text{--- (31)}$$

$\therefore V_e$ when $\boxed{Z_s = 0}$ & $\boxed{Z_{sh} = \infty}$ — (32)

subs (32) in (31)

ie
$$V_e = \frac{E(s)\left[\cosh PL + \frac{Z_0}{Z_t}\sinh PL\right]}{[1+0+0]\cosh PL + \left[\frac{Z_0}{Z_t}+0+0\right]\sinh PL}$$

$$\therefore V_e = \frac{E(s)\left[\cancel{\cosh PL + \frac{Z_0}{Z_t}\sinh PL}\right]}{\left[\cancel{\cosh PL + \frac{Z_0}{Z_t}\sinh PL}\right]}$$

$$\boxed{\therefore V_e = E(s)} \quad \text{--- (33)}$$

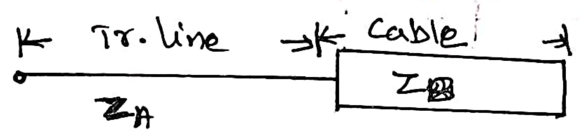
from (33) the voltage at the entrance of the line V_e becomes equal to the source voltage $E(s)$

— x — x — x — x — x.

Reflection and Refraction of Travelling waves:-

When a travelling wave on a transmission line reaches a transition point at which there is an abrupt change of line parameters, as open or short circuit termination, a junction with another line, a machine winding, load termination etc.,

* A part of wave is reflected back on the incoming line and the rest may pass through other line section.



Derivation of Reflection and Refraction co-efficient:-

* The travelling wave before reaching a transition point is called the incident wave.

* The incident wave may be decomposed into two component waves called.

(i) Reflected wave

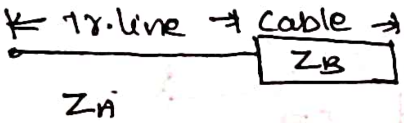
(ii) Transmitted wave (or) Refracted wave

* Consider the junction between lines of characteristic impedance (Z_A) and Z_B and let us suppose that 'Z_A' > 'Z_B'

* For example this might be be junction between an overhead line and cable.

* Suppose that a voltage surge of step function form and amplitude 'V_1' approaches the junction along the overhead line.

* The current wave will have the same shape and amplitude



Let ~~sep~~ subscripts. 1, 2 & 3 for incident wave.

Let subscripts

1 → for Incident wave

2 → for Reflected wave

3 → for Refracted wave

$$\therefore \boxed{I_1 = \frac{V_1}{Z_A}} \quad \text{-----------} (1)$$

* Let the scattered and refracted voltages waves be V_2 and V_3 respectively, so that their current will be

$$\boxed{I_2 = \frac{-V_2}{Z_A}} \quad \text{———} \quad (2)$$

$$\boxed{I_3 = \frac{V_3}{Z_B}} \quad \text{———} \quad (3).$$

* In equation number (2) minus sign indicates that the wave travelling in a direction opposite to that of incident wave

* If voltage and current are to be continuous at the junction it follows that

$$\boxed{V_1 + V_2 = V_3} \quad \text{.........} (4)$$

$$\boxed{I_1 + I_2 = I_3} \quad \text{.........} (5)$$

Sub equation number (1), (2) & (3) in equation no:(5)

$$\boxed{\frac{V_1}{Z_A} - \frac{V_2}{Z_A} = \frac{V_3}{Z_B}} \quad \text{———} \quad (6)$$

Substitute equation no:-4 in equation no:-6

$$\therefore \quad \frac{V_1}{Z_A} - \frac{V_2}{Z_A} = \frac{V_1 + V_2}{Z_B}$$

$$\frac{V_1}{Z_A} - \frac{V_2}{Z_A} = \frac{V_1}{Z_B} + \frac{V_2}{Z_B}$$

Rearranging the above equation

$$\frac{V_1}{Z_A} - \frac{V_1}{Z_B} = \frac{V_2}{Z_B} + \frac{V_2}{Z_A}$$

$$\therefore V_1 \left[\frac{1}{Z_A} - \frac{1}{Z_B} \right] = V_2 \left[\frac{1}{Z_B} + \frac{1}{Z_A} \right]$$

$$V_1 \left[\frac{Z_B - Z_A}{\cancel{Z_A} Z_B} \right] = V_2 \left[\frac{Z_B + Z_A}{Z_B \cancel{Z_A}} \right]$$

$$\therefore V_2 = \left[\frac{Z_B - Z_A}{Z_B + Z_A} \right] V_1$$

$$\boxed{\therefore V_2 = \left[\frac{Z_B - Z_A}{Z_B + Z_A} \right] V_1} \qquad \text{------} \quad (7)$$

where $V_2 \rightarrow$ Reflected wave interms of Incident wave similarly refracted wave can be written in terms of V_1 as

* The quantity $\left[\dfrac{Z_B - Z_A}{Z_B + Z_A}\right]$ is called the reflection co-efficient (α)

* The α value may vary from $-1 \leq \alpha \leq +1$

$$\therefore \boxed{V_2 = \alpha V_1} \quad \text{———} (8)$$

Refracted wave interms of Incident wave:-

(4) $\Rightarrow$ $V_1 + V_2 = V_3$

$$\boxed{\therefore V_2 = V_3 - V_1} \quad \text{———} (9)$$

(6) $\Rightarrow$ $\dfrac{V_1}{Z_A} - \dfrac{V_2}{Z_A} = \dfrac{V_3}{Z_B}$

sub (9) in (6)

$$\frac{V_1}{Z_A} - \frac{(V_3 - V_1)}{Z_A} = \frac{V_3}{Z_B}$$

$$\frac{V_1}{Z_A} - \frac{V_3}{Z_A} + \frac{V_1}{Z_A} = \frac{V_3}{Z_B}$$

$$\frac{V_3}{Z_B} + \frac{V_3}{Z_A} = \frac{2V_1}{Z_A}$$

$$V_3\left[\frac{1}{Z_B}+\frac{1}{Z_A}\right]=\frac{2V_1}{Z_A}$$

$$V_3\left[\frac{Z_B+Z_A}{Z_B Z_A}\right]=\frac{2V_1}{Z_A}$$

$$\boxed{\therefore\ V_3=\left[\frac{2Z_B}{Z_B+Z_A}\right]V_1} \quad \text{--- (10)}$$

$$\boxed{V_3 = bV_1} \quad \text{--- (11)}$$

where $b \rightarrow$ Refraction co-efficient

ie $$\boxed{b=\frac{2Z_B}{Z_B+Z_A}} \quad \text{--- (12)}$$

The value of b varies between 0 ~~to~~ and two

$\therefore$ $0 \leq b \leq 2$ depending upon the relative value of Z_A & Z_B.

* The reflected and refracted wave can be shown as figure by considering the following Example

Example:-

suppose that $Z_A = 400\,\Omega$ and $Z_B = 50\,\Omega$

and that $V_1 = 300\,kV$

then $I_1 = \frac{V_1}{Z_A}$

ie $I_1 = \frac{300 \times 10^3}{400}$

$\therefore \boxed{I_1 = 750\,A}$ —(13)

$\therefore \alpha = \frac{Z_B - Z_A}{Z_B + Z_A}$

$\alpha = \frac{50 - 400}{50 + 400} = \frac{-350}{450}$

$\boxed{\alpha = -0.78}$ —(14)

where $\alpha \rightarrow$ Reflection co-efficient

W.K.T $V_2 = \alpha V_1$

$\therefore V_2 = -0.78 \times 300 \times 10^3$

$\boxed{V_2 = -234\,kV}$ —(15)

W.K.T $b = \frac{2 Z_B}{Z_B + Z_A}$

$b = \frac{2 \times 50}{50 + 400}$

$$\therefore \boxed{b = 0.22} \quad \text{--- (16)}$$

where $b \rightarrow$ refracted co-efficient.

$$\therefore \quad V_3 = b V_1$$

$$V_3 = 0.22 \times 300 \times 10^3$$

$$\boxed{V_3 = 66\,kV} \longrightarrow \text{Refracted voltage wave}$$

$$\boxed{\therefore V_3 = 66\,kV} \text{ --- (17)}$$

similarly $\boxed{I_2 = \frac{-V_2}{Z_A}}$ --- (18) sub (15) in (18)

$$= \frac{+234 \times 10^3}{400}$$

$$\boxed{I_2 = +585\,A} \text{ --- 19}$$

$$I_3 = \frac{V_3}{Z_B}$$

$$= \frac{66 \times 10^3}{50}$$

$$\boxed{\therefore I_3 = 1320\,A} \text{ --- (20)}$$

where V_1 & $I_1 \longrightarrow$ ~~Refracte~~ Incident voltage & current wave

V_2 & $I_2 \longrightarrow$ Reflected voltage & current wave

V_3 & $I_3 \longrightarrow$ Refracted voltage & current wave

$V_1 = 300\,kV \qquad I_1 = 750\,A,$

$V_2 = -234\,kV \qquad I_2 = -585\,A$

$V_3 = 66\,kV \qquad I_3 = 1320\,A$

These can be illustrated in following fig.

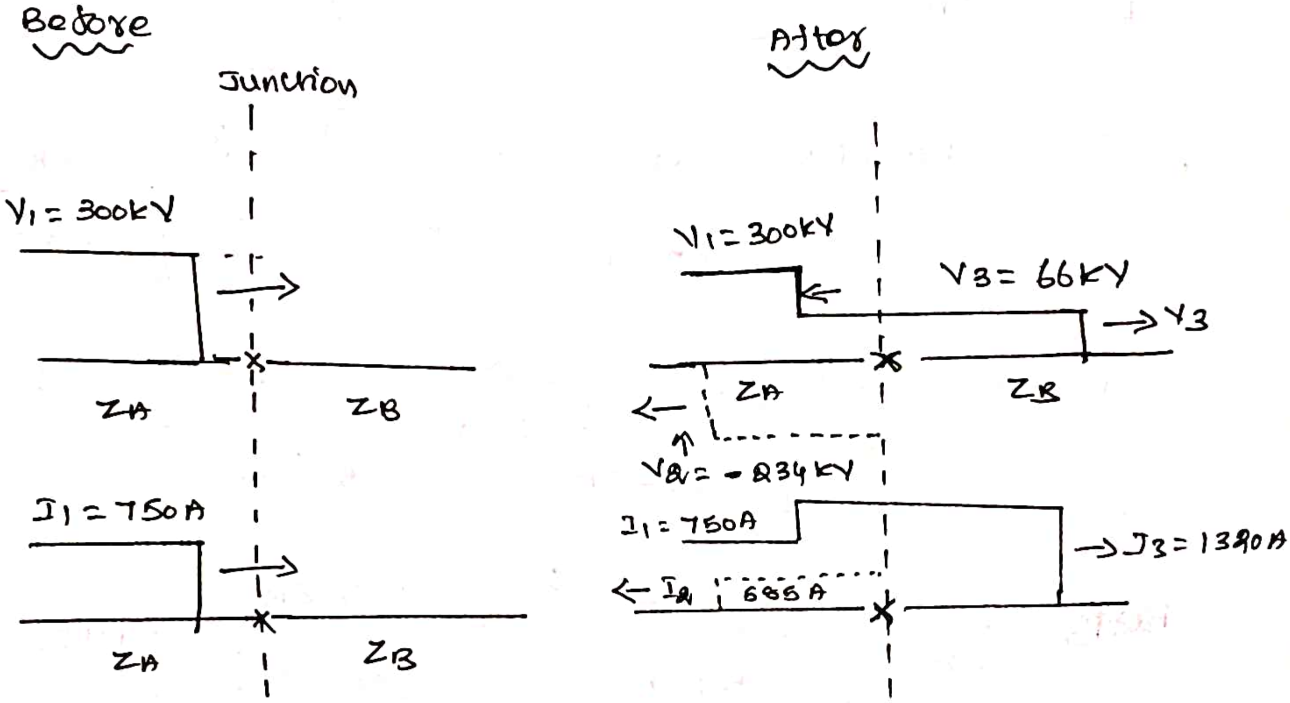


fig:- Voltage and current waves being reflected and refracted at a junction between two lines

Energy conservation:-

The energy supplied by the source is

$$\boxed{P_1 = \frac{V_1^2}{Z_A} \text{ Watts}} \quad \text{--- (21)}$$

* When this energy reaches the junction, it is being dispatched from the junction by the reflected and refracted waves at a rate

$$\boxed{P_2 = \frac{V_2^2}{Z_A} \;;\; P_3 = \frac{V_3^2}{Z_B} \text{ watts}} \quad \text{—— (22)}$$

$$\therefore \; P_2 + P_3 = \frac{V_1^2}{Z_A}\left[\frac{Z_B - Z_A}{Z_B + Z_A}\right]^2 + \frac{V_1^2}{Z_B}\left[\frac{2Z_B}{Z_B + Z_A}\right]^2$$

$$\therefore \; P_2 + P_3 = \frac{V_1^2}{Z_A}\left[\left(\frac{Z_B - Z_A}{Z_B + Z_A}\right]^2 + \frac{4 Z_A Z_B^{\cancel{2}}}{\cancel{Z_B}\,(Z_B + Z_A)^2}\right]$$

$$P_2 + P_3 = \frac{V_1^2}{Z_A}\left[\left(\frac{Z_B - Z_A}{Z_B + Z_A}\right)^2 + \frac{4 Z_A Z_B}{(Z_B + Z_A)^2}\right] \text{ watts}$$

—— (23)

* The quantity inside the bracket reduces to unity so that the reflected and refracted waves contain the same energy as the incident wave.

Reflected and Refraction of Travelling waves

Reflation and Refraction of travelling waves - when the line is divided into another lines :-

when a line joins two or more other lines the refracted waves be :-

$$\boxed{I_{3B} = \frac{V_{3B}}{Z_B}}, \quad \boxed{I_{3C} = \frac{V_{3C}}{Z_C}}$$

for 'N' no. of lines

$$\boxed{I_{3N} = \frac{V_{3N}}{Z_N}} \quad \text{--- (24)}$$

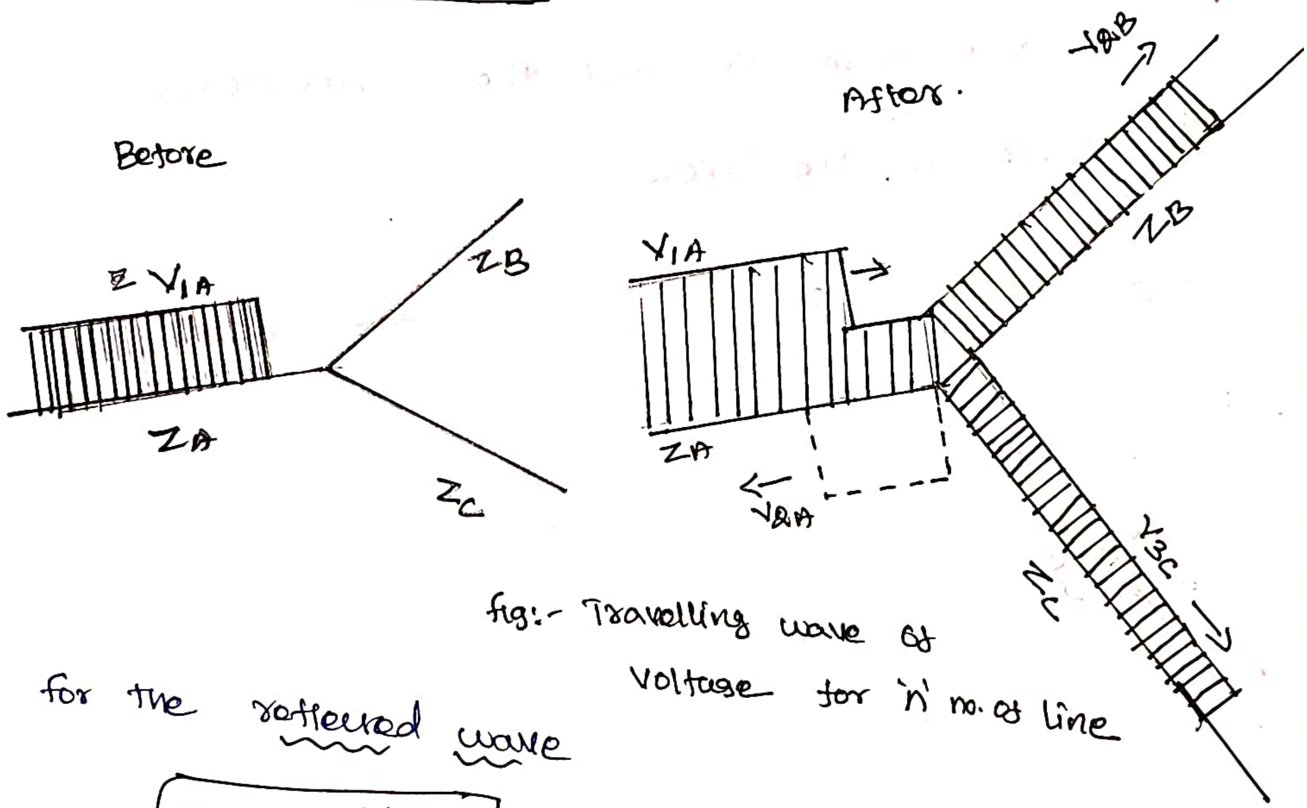


fig:- Travelling wave of Voltage for 'n' no. of line

* for the reflected wave

$$\boxed{I_{2A} = \frac{-V_{2A}}{Z_A}} \quad \text{--- (25)}$$

For continuity of voltage

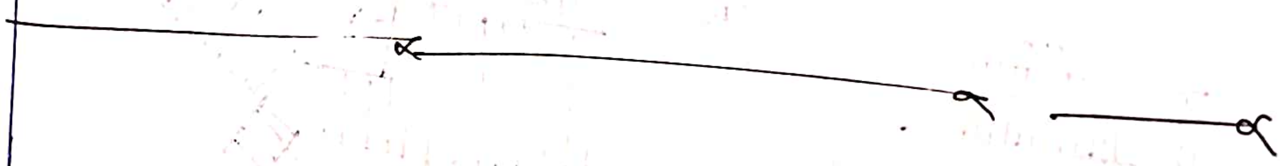
$$\cancel{V_{1A} + V_{2A} =}$$

$$V_{1A} + V_{2A} = V_{3B} = V_{3C} = \cdots\cdots = V_{3N} \quad \text{--- (26)}$$

& For continuity of current

$$\left.\begin{aligned} I_{1A} + I_{2A} &= I_{3B} + I_{3C} + \cdots\cdots + I_{3N} \\ \text{ie} \quad I_{1A} + I_{2A} &= I_{3B} + I_{3C} + \cdots\cdots + I_{3N}. \end{aligned}\right\} 27$$

Equation no:- 24 to 27 are sufficient to specify reflected wave and all the refracted waves interms of incident wave V_{A1} and the characteristics impedance of the lines.



BEHAVIOR OF TRAVELLING WAVES AT LINE TERMINATIONS:-

Another obvious type of line discontinuity is the line termination. There are two types of terminations

(i) Short circuit

(ii) Open circuit.

(i) Short circuit:-

* The unique characteristics of the short circuit is that it is impossible to develop any voltage across it.

* Thus when a travelling wave of voltage reaches a short circuit, the reflected voltage wave cancel out the incident wave so that the refracted wave is zero.

* If the incident voltage wave is V_i and the incident current wave I_1 the reflected voltage wave will be $-V_1$ and the reflected current wave $+ I_R (= I_1)$. This is illustrated in fig.

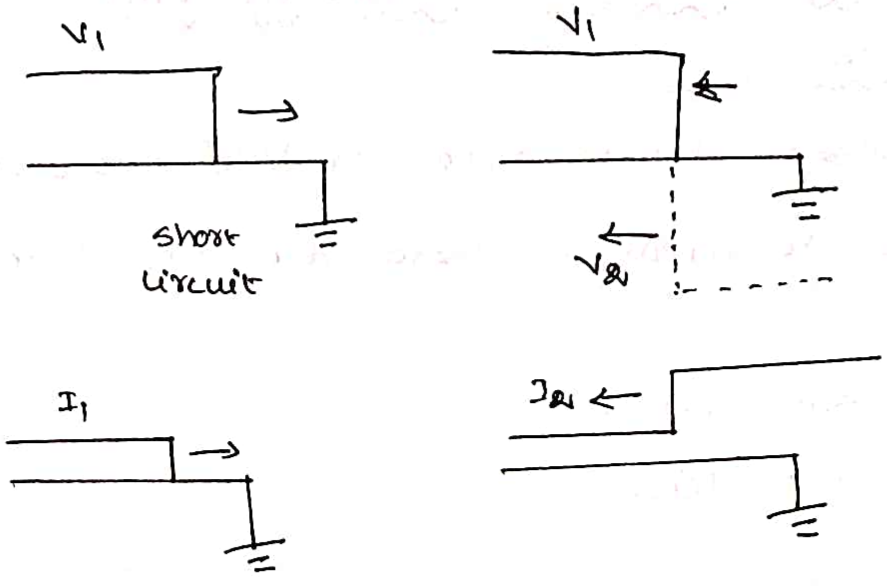


Fig:- Reflection of travelling wave from a short circuit.

* The total current at the junction point

$$I_2 + I_1 = 2I$$

* Thus the current at the junction point rises to double the value of incident current wave.

When fed by a voltage source:-

* When a short circuit is applied to a transmission line fed by a voltage source, the fault current will increase indefinitely at a rate of '

$$\boxed{\frac{V}{LL}}$$

where $LL \rightarrow$ Inductance of fault point L metres away.

fig (a)

* When the short circuit is applied a wave of voltage magnitude $-V$, travels toward the source reducing the line voltage to zero.

* The accompanying current wave is $\frac{+V}{Z_0}$. This is shown in fig 'a'.

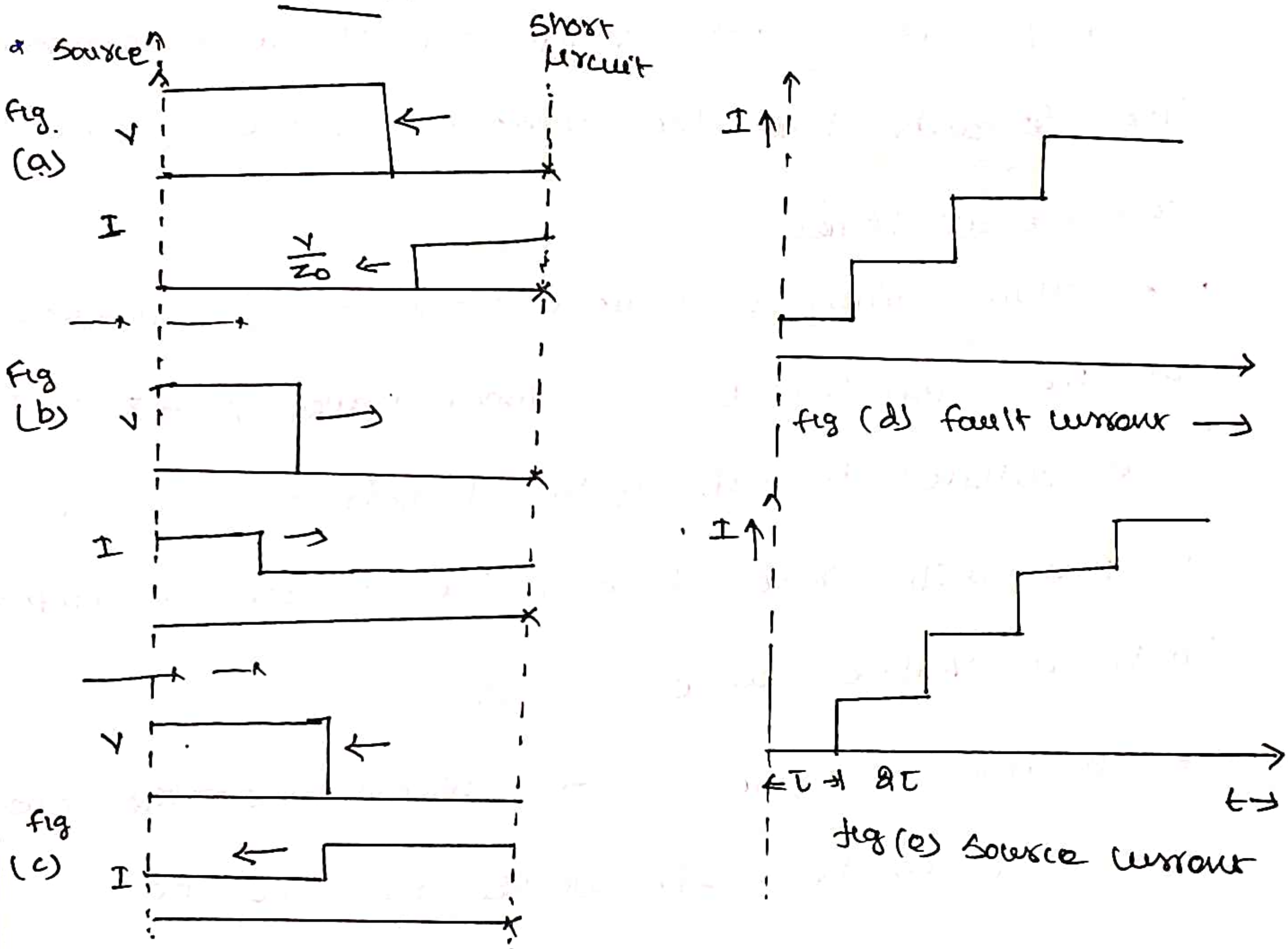


Fig:- Buildup of current when a short circuit occurs on Tr. line

fig (b)

* When this wave reaches the source, the boundary condition there demands the initiation of new wave of voltage $+V$, with a current $+\frac{V}{Z_0}$

fig(c):- cycle repeats

fig(d):- & fig(e)

* The fault & source current increases in discrete step.

(ii) Open circuit:-

* An open circuit at the end of a transmission line demands that the current at that point be zero at all times.

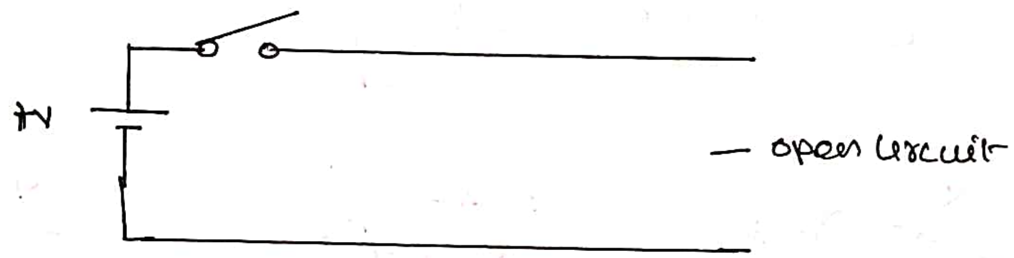
* Thus when a current wave of + I arrives at the open circuit a current wave of − I is at once initiated to satisfy the boundary condition.

* This will travel toward the source in company with a voltage wave of +V.

* A current wave of − I incident on the open circuit would be reflected as + I and be associated with − V.

* what happens when a open circuited line is energized from a source of 'V' volts is shown in fig.

* In this situation the magnetic energy associated with current disappears when the current is brought to zero at the open circuit & it reappears as the electric energy which manifests itself in the doubling of voltage.



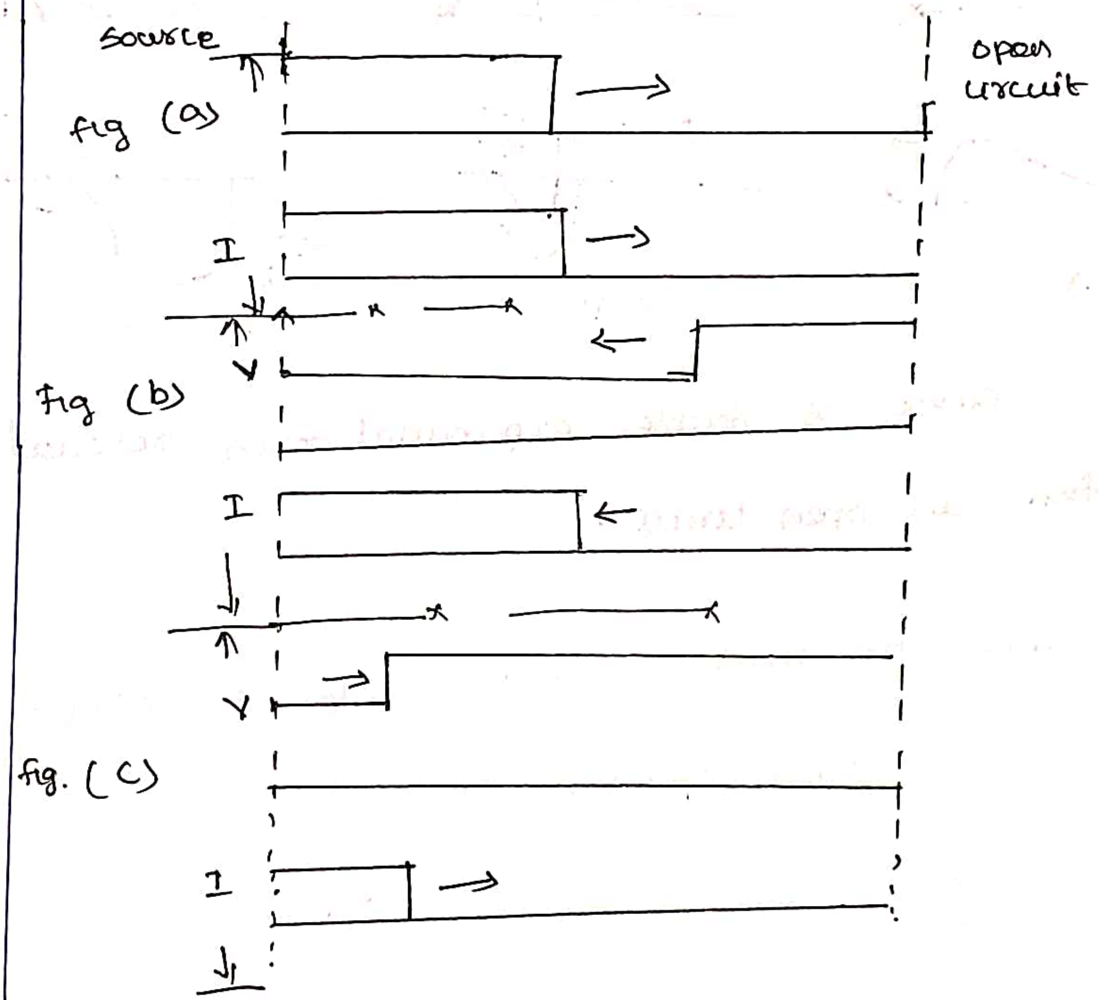


fig:- Travelling waves initiated by energizing an open circuited line

* ~~In the~~ So far we have been concerned with travelling waves of step-function form.

* A surge waveform which is closely approximated by many practical surges is the double exponential

$$\boxed{V = V_0 \left(e^{-\alpha t} - e^{-\beta t}\right)}$$

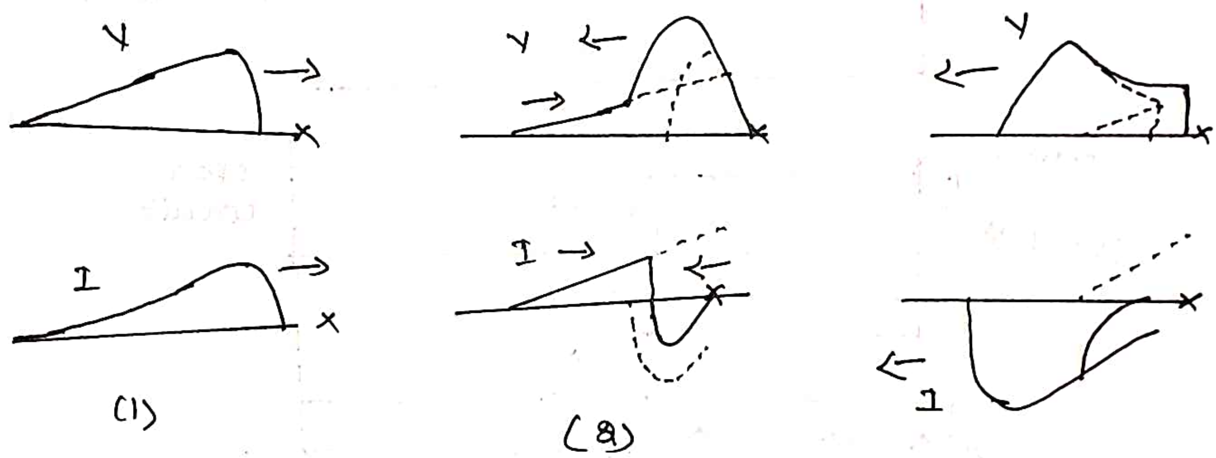


fig:- Surge of double exponential form reflected from an open circuit.

* Fig shows how ~~reflect~~ such a wave is reflected from an open circuit.

GENERAL TERMINATIONS:-

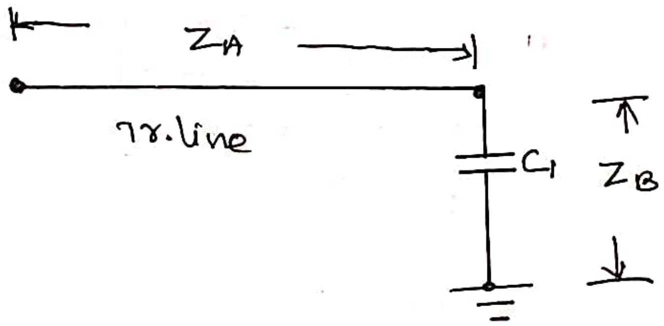
There are generally three terminations namely

(i) Line Termination through Capacitance

(ii) Line termination through Inductance

(iii) Line termination through Resistance.

(i) Line termination through Capacitance:-



Let Z_A as Transmission line Impedance

$Z_B \rightarrow$ Impedance of Capacitor $\therefore Z_B = \frac{1}{C_1}$

$V_1 \rightarrow$ Incident Voltage

$V_2 \rightarrow$ Reflected Voltage

$V_3 \rightarrow$ Refracted Voltage (Transmitted voltage)

Calculation of Reflected Voltage (V_2)

W.K.T

~~$V_{2r}(s) =$~~ $V_2(s) = \alpha V_1(s)$ —— (1)

If the travelling wave is a step function of amplitude V_1 then $\boxed{V_1(s) = \frac{V_1}{s}}$ —— (2)

W.K.T $a = \frac{Z_B - Z_A}{Z_B + Z_A}$ —(3)

$\therefore \quad a = \frac{\frac{1}{C_1} - Z_A}{\frac{1}{C_1} + Z_A}$ $\quad \left[L\left[\frac{1}{C_1}\right] = \frac{1}{C_1 s}\right]$

$\therefore \quad V_{R}(s) = \frac{V_1}{s} \left[\frac{\frac{1}{C_1 s} - Z_A}{\frac{1}{C_1 s} + Z_A}\right]$ [sub (2) & (3) in (1) & take Laplace]

$$V_{R}(s) = \frac{V_1}{s}\left[\frac{\frac{1 - C_1 s Z_A}{\cancel{C_1 s}}}{\frac{1 + C_1 s Z_A}{\cancel{C_1 s}}}\right]$$

$$= \frac{V_1}{s}\left[\frac{1 - C_1 s Z_A}{1 + C_1 s Z_A}\right]$$

$$= \frac{V_1}{s} \frac{C_1 \cancel{Z_A}}{C_1 \cancel{Z_A}}\left[\frac{\frac{1}{C_1 Z_A} - s}{\frac{1}{C_1 Z_A} + s}\right]$$

$$\therefore V_{R}(s) = \frac{V_1}{s}\left[\frac{\frac{1}{C_1 Z_A} - s}{\frac{1}{C_1 Z_A} + s}\right]$$

$$V_0(s) = \frac{V_1}{s}\left[\frac{\frac{1}{C_1 Z_A} - s}{\frac{1}{C_1 Z_A} + s}\right]$$

Let $\frac{1}{C_1 Z_A} = \alpha$

$$\therefore V_0(s) = \frac{V_1}{s}\left[\frac{\alpha - s}{\alpha + s}\right]$$

$$= \frac{V_1}{s}\left[\frac{\alpha}{\alpha + s} - \frac{s}{\alpha + s}\right]$$

$$V_0(s) = V_1\left[\frac{\alpha}{s(s+\alpha)} - \frac{1}{s+\alpha}\right] \text{ ——— } (4)$$

Apply partial fraction for $\frac{\alpha}{s(s+\alpha)}$

$$\therefore \frac{\alpha}{s(s+\alpha)} = \frac{A}{s} + \frac{B}{s+\alpha} \text{ —— } (5)$$

$$\therefore \frac{\alpha}{s(s+\alpha)} = \frac{A(s+\alpha) + Bs}{s(s+\alpha)}$$

$$\therefore \alpha = A(s+\alpha) + Bs \text{ ——— } (6)$$

put $\boxed{s=0}$ in eqn (5)

$$\therefore \alpha = A\alpha + 0$$

$$\boxed{A=1} \text{ - - - - - - - - } (7)$$

put $\boxed{s=-\alpha}$

$$\alpha = 0 - B\alpha$$

$$B\alpha = -\alpha$$

$$\boxed{B=-1}$$

sub (A) & (B) value in eqn(6)

$$\boxed{\frac{\alpha}{s(s+\alpha)} = \frac{1}{s} - \frac{1}{s+\alpha}} \quad \text{—— (8)}$$

sub(8) in eqn(4)

$$\xrightarrow{\alpha} \quad V_R(s) = V_1\left[\frac{1}{s} - \frac{1}{s+\alpha} - \frac{1}{s+\alpha}\right] \quad \text{—— (9)}$$

Take Inverse Laplace Transform for eqn(9).

$$V_R(t) = V_1\left[1 - e^{-\alpha t} - e^{-\alpha t}\right]$$

$$\boxed{\therefore V_R(t) = V_1\left[1 - 2e^{-\alpha t}\right]} \quad \text{—— (10)}$$

Equation (10) gives Reflected voltage wave

Calculation of ~~Reflected~~ Refracted Voltage (V_3):-

W.K.T $\boxed{V_3 = V_2 + V_1}$ —— (11)

Sub eqn no. (10) in equation number (11)

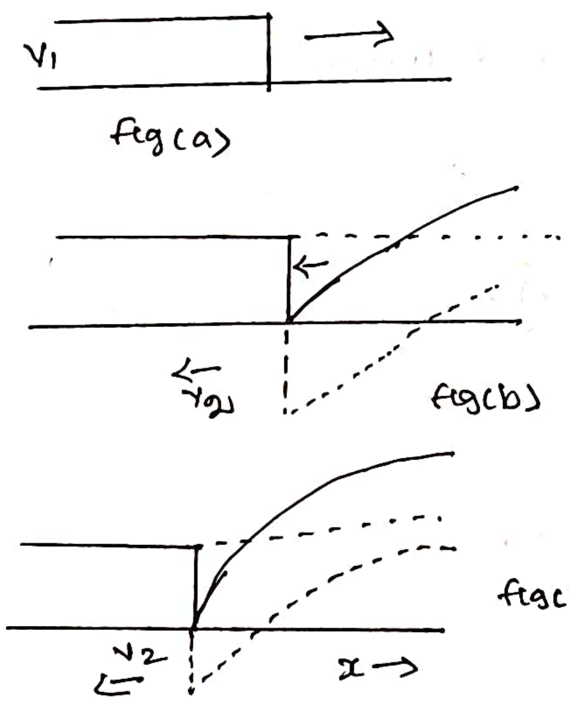
$$V_3(t) = V_1 + V_1\left[1 - 2e^{-\alpha t}\right]$$

$$V_3(t) = V_1 + V_1 - 2V_1 e^{-\alpha t}$$

$$V_3(t) = 2V_1 - 2V_1 e^{-\alpha t}$$

$$\boxed{V_3(t) = V_1\left[2 - 2e^{-\alpha t}\right]} \quad \text{—— (12)}$$

Equation (12) gives refracted (or) transmitted voltage wave



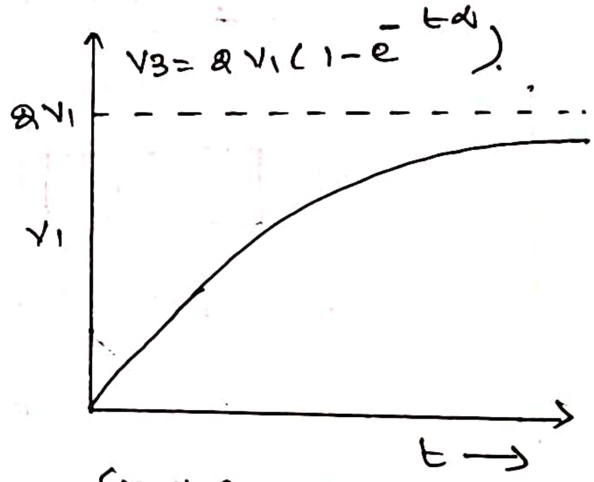
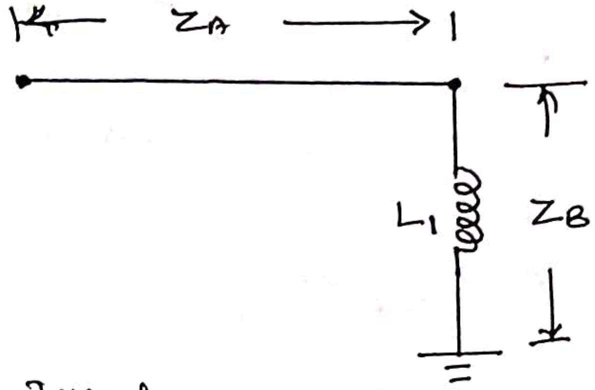


Fig: (d) Capacitor terminal voltage

fig (a),(b),(c) — Dis position of the wave at different instances.

(ii) Line termination through Inductance:



Let $Z_A \rightarrow$ Impedance of Tr. Line

$Z_B \rightarrow$ Impedance of inductor 'L',

$$\boxed{Z_B = L_1 S}$$

Calculation of Reflected voltage :- (V_R)

W.R.T eq ① $\Rightarrow V_R(S) = a V_1(S)$ ——

$$a = \frac{Z_B - Z_A}{Z_B - Z_A}$$ ——

sub $\boxed{Z_B = L_1 S}$ in above equation

$$\therefore a = \frac{L_1 S - Z_A}{L_1 S + Z_A}$$

$$\therefore V_1(S) = \frac{V_1}{S}$$

(13)

sub (13) in equation number (1)

D

$$V_Q(s) = \frac{V_1}{s}\left[\frac{L_1 s - Z_A}{L_1 s + Z_A}\right]$$

$$= \frac{V_1}{s} \times \frac{\cancel{L_1}}{\cancel{L_1}}\left[\frac{s - \frac{Z_A}{L_1}}{s + \frac{Z_A}{L_1}}\right]$$

$$V_Q(s) = \frac{V_1}{s}\left[\frac{s-\beta}{s+\beta}\right]$$

$$= \frac{V_1}{s}\left[\frac{s}{s+\beta} - \frac{\beta}{s+\beta}\right] \qquad \left[\text{Let } \beta = \frac{Z_A}{L_1}\right]$$

$$V_Q(s) = V_1\left[\frac{1}{s+\beta} - \frac{\beta}{s[s+\beta]}\right] \quad \text{—— (14)}$$

Apply partial fraction for $\frac{\beta}{s(s+\beta)}$ term

$$\frac{\beta}{s[s+\beta]} = \frac{A}{s} + \frac{B}{s+\beta}$$

$\therefore \beta = A(s+\beta) + Bs$ put $\boxed{s=0}$

$\therefore \beta = A\beta + 0$

$\therefore \boxed{A=1}$ put $\boxed{s=-\beta}$

$\therefore \beta = 0 - \beta B$

$\therefore \boxed{B=-1}$

$$\therefore \frac{\beta}{s(s+\beta)} = \frac{1}{s} - \frac{1}{s+\beta} \quad \text{——— (15)}$$

sub(15) in (14)

$$V_2(s) = V_1 \left[\frac{1}{s+\beta} - \frac{1}{s} + \frac{1}{s+\beta} \right] \quad \text{——— (16)}$$

$V_2(s)$ Take Inverse laplace of eqn (16)

$$V_2(t) = \cancel{V_1} \left[e^{-\beta t} \cancel{+} e \right.$$

$$V_2(t) = V_1 \left[e^{-\beta t} - 1 + e^{-\beta t} \right]$$

$$\boxed{V_2(t) = -V_1 \left[1 - 2e^{-\beta t} \right]} \quad \text{——— (17)}$$

Equation no.(17) Reflected voltage wave

Calculation of Refracted voltage:- (V_3)

$$V_3(t) = V_1 + V_2$$

$$\cancel{V_3(t) = V_1 - V_1 \left[1 - 2e^{-\beta t} \right]}$$

$$V_3(t) = V_1 - V_1 \left[1 - 2e^{-\beta t} \right]$$

$$V_3(t) = \cancel{V_1} - \cancel{V_1} + 2V_1 e^{-\beta t}$$

$$\boxed{\therefore V_3(t) = 2V_1 e^{-\beta t}} \quad \text{——— (18)}$$

(e)

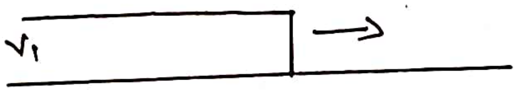
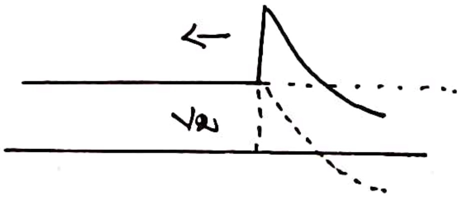


fig (a)



fig(b)

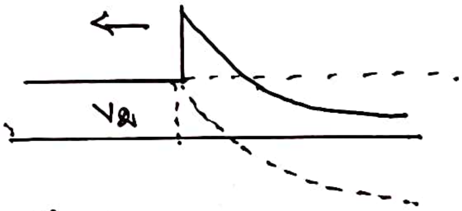
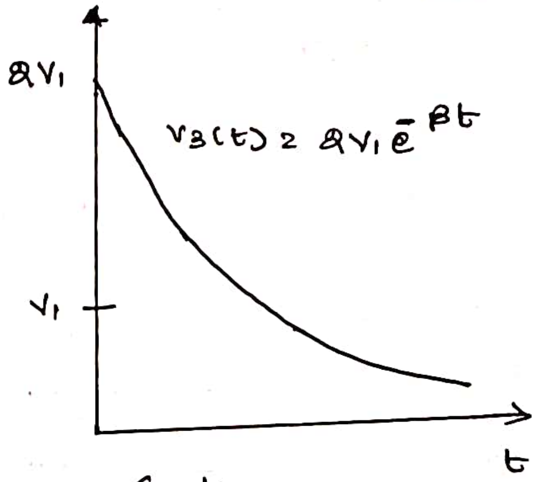


fig (c)

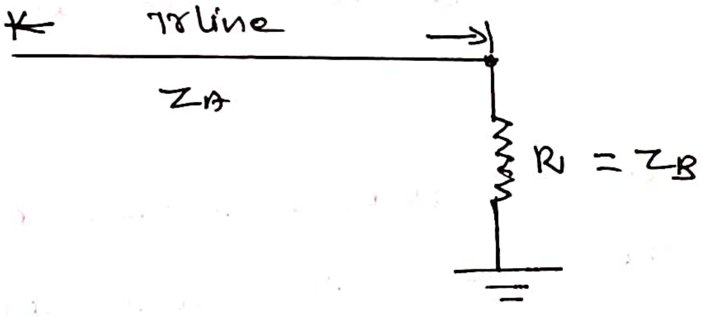


fig(d)

figs (a),(b) (c) Disposition of the wave at different instants

fig (d) Inductor terminal voltage.

(iii) Line termination through Resistance:-



Let Z_B is a resistor numerically equal to the Line Impedance Z_A $\therefore$ $\boxed{Z_A = Z_B}$

$\therefore$ Refracted voltage $V_3 = \left[\dfrac{2 Z_B}{Z_B + Z_A}\right] V_1$

$$\therefore V_3 = \left[\frac{2 Z_B}{2 Z_B}\right] V_1$$

$$\therefore \boxed{V_3 = V_1}$$

* This means that incident wave is completely absorbed. There is no reflected wave.

Note:- Refer page 250 Allan Greenwood

(i) Capacitor (C) cannot instantaneously change its potential. $\therefore$ capacitor behaves like a short circuit.

* Capacitor represents no path for direct current. In this respect it is like an open circuit.

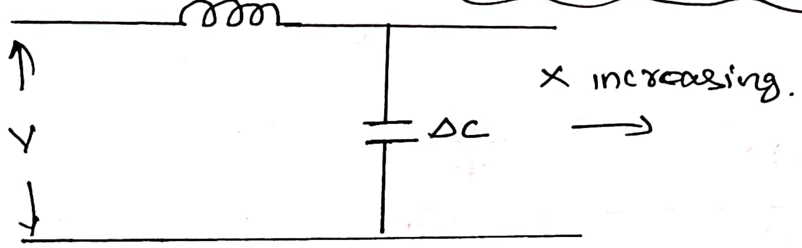
(ii) Inductor:-

(a) No current can penetrate instantaneously when the wave arrives at its terminals momentarily its appear like an open circuit.

(b) Pure inductance presents no impedance to a direct current, so it appears like as a short circuit.

TRAVELLING WAVE - STEP RESPONSE

- Consider a lossless two wire line
- The fig shows a small element of a transmission lines.



{fig: small part of Tr. line.

- If the line has an inductance of L henries per meter and a capacitance of 'C' farads per meter and an elementary length Δx will have inductance $L\Delta x$ and $C\Delta x$ as shown in fig.
- The voltage across the element will be

$$-\Delta V = L\cdot\Delta x\,\frac{\partial I}{\partial t} \qquad \text{——— (1)}$$

$\therefore$

$$\frac{\Delta V}{\Delta x} = -L\cdot\frac{dI}{dt}$$

$$\boxed{\frac{\partial V}{\partial x} = -L\cdot\frac{dI}{dt}} \qquad \text{——— (2)}$$

* Here the partial derivatives are used because V & I are functions of both position and time.

* The current to charge the elementary capacitance ΔC is given by

$$-\Delta I = C \Delta x \frac{\partial V}{\partial t}$$

$$\frac{\Delta I}{\partial x} = -C \cdot \frac{\partial V}{\partial t}$$

$$\boxed{\frac{\partial I}{\partial x} = -C \cdot \frac{\partial V}{\partial t}} \quad \text{———} \quad (3)$$

* Now I can be eliminated from the pair of simultaneous equations by differentiating equation equation (2) w.r. to 'x' and Equation (3) w.r. to 't'

$$(2) \Rightarrow \boxed{\frac{\partial^2 V}{\partial x^2} = -L \frac{\partial^2 I}{\partial x \partial t}} \quad \text{———} \quad (4)$$

$$(3) \Rightarrow \boxed{\frac{\partial^2 I}{\partial x \, \partial t} = -C \frac{\partial^2 V}{\partial t^2}} \quad \text{———} \quad (5)$$

sub (5) in (4)

$$\boxed{\frac{\partial^2 V}{\partial x^2} = LC \frac{\partial^2 V}{\partial t^2}} \text{ ———— } (6)$$

→ (Equation for voltage)

Now for current 'I' diff. eqn (2) w.r. to 't' and eqn (3) w.r. to 'x'

$$\boxed{\frac{\partial^2 V}{\partial x \partial t} = -L \cdot \frac{\partial^2 I}{\partial t^2}} \text{ ———— } (7)$$

$$(3) \Rightarrow \boxed{\frac{\partial^2 I}{\partial x^2} = -C \frac{\partial^2 V}{\partial x \partial t}} \text{ ———— } (8)$$

sub (7) in (8)

$$\boxed{\frac{\partial^2 I}{\partial x^2} = LC \frac{\partial^2 I}{\partial t^2}} \text{ ———— } (9)$$

(Equation for current)

⇒ Equation (6) & (9) constitutes wave equation.

Consider the Equation for current:- (Equation No:-9)

The solution of current Equation is given by

$$I = f\left(x \mp \frac{t}{(LC)^{1/2}}\right) \text{ ———— } (10)$$

(or)

$$I = f(t \mp$$

($\gamma \rightarrow$ Propagation constant for voltage)

$$\boxed{I = f(t \mp (LC)^{1/2} x)} \quad \text{———} \quad (4)$$

⇒ Equation (10) states that 'I' is function of 'x' and equation (11) states that 'I' is function of 't'

Equation No:-(10) can be written as

$$\boxed{I(x, t) = f_1(x - \gamma t) + f_2(x + \gamma t)} \quad \text{———} \quad (12)$$

* This is the solution of equation no:-(9)

Solution of voltage

* The solution of voltage can be obtained from equation number (8)

$$(8) \Rightarrow \frac{\partial V}{\partial x} = -L \cdot \frac{\partial I}{\partial t} \quad \text{———} \quad (13)$$

Diff equation No:-(12) w.r.to, 't' and substitute in equation number (13)

$$\frac{\partial I}{\partial t} = -\gamma f_1'(x - \gamma t) + f_2' \gamma (x + \gamma t)$$

$$=$$

$$\frac{\partial V}{\partial x} = -L \cdot \frac{\partial I}{\partial t}$$

$$= -L\left[-v f_1'(x-vt) + v f_2'(x+vt)\right]$$

$$\frac{\partial V}{\partial x} = Lv\left[f_1'(x-vt) - f_2'(x+vt)\right]$$

$$= L \cdot \frac{1}{\sqrt{LC}}\left[f_1'(x-vt) - f_2'(x+vt)\right] \quad \left[\because v = \frac{1}{\sqrt{LC}}\right.$$

$$\therefore \frac{\partial V}{\partial x} = \frac{\sqrt{L} \cdot \cancel{\sqrt{L}}}{\cancel{\sqrt{L}} \cdot \sqrt{C}}\left[f_1'(x-vt) - f_2'(x+vt)\right]$$

$$\frac{\partial V}{\partial x} = \sqrt{\frac{L}{C}}\left[f_1'(x-vt) - f_2'(x+vt)\right]$$

$$\frac{\partial V}{\partial x} = Z_0\left[f_1'(x-vt) - f_2'(x+vt)\right] \quad \text{—— (14)}$$

$$\left[\because Z_0 = \sqrt{\frac{L}{C}}\right]$$

Integrated eqn no:- (14) w. r. to x,

$$\boxed{\cancel{V(x,t)} = \cancel{Z_0}\left[\cancel{f_1}(x-vt) - \cancel{f_2}(\cancel{x}+vt)\right]} \quad \text{—— } \cancel{(15)}$$

Eqn (12) $\Rightarrow$ I

Integrating on both sides of equation no: 14

w.r.to 'x'

$$\boxed{V(x,t) = Z_0\left[f_1(x-vt) + f_2(x+vt)\right]} \quad -(15)$$

Equation no:-(12)

$$\boxed{I(x,t) = f_1(x-vt) + f_2(x+vt)} \quad -(12)$$

comparing equation no:- (12) & (15)

$$\therefore \quad I(x,t) = \frac{V(x,t)}{Z_0},$$

we note that direct proportionality between voltage and current the proportionality factor being the characteristic Impedance (Z_0).

Consider the function $f_1(x-vt)$

* At time (i) $\boxed{t=0}$ It has a spatial distribution $f_1(x)$ and a value at $\boxed{x=a}$ of $\boxed{f_1(a)}$

* At any subsequent time (ii) $\boxed{t=\tau}$, it has the same value at $\boxed{x=(a+v\tau)}$ as it formerly had at $x=a$

* Which says that the distribution has moved in fact a distance $v\tau$ in the direction of plus x. This is illustrated in fig.

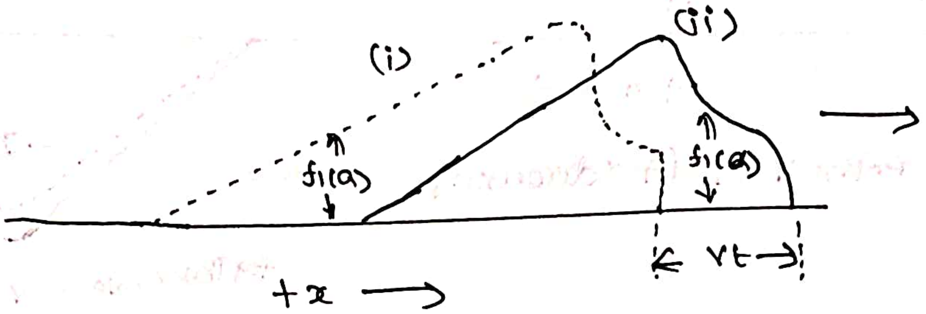


fig:- The function $(x - vt)$ at (i) $t = 0$ & (ii) $(t = \tau)$

similarly the function $f_2(x + vt)$

* If represents a distribution moving in a direction of minus 'x' with a velocity 'v'

fig:- for function $f_2(x + vt)$ at (i) $t = 0$ (ii) $t = \tau$

* we also note that current and voltage waves travelling in the positive direction of x have the same sign, whereas those travelling the negative direction have opposite sign.

Different combinations of voltage and current wave forms:-

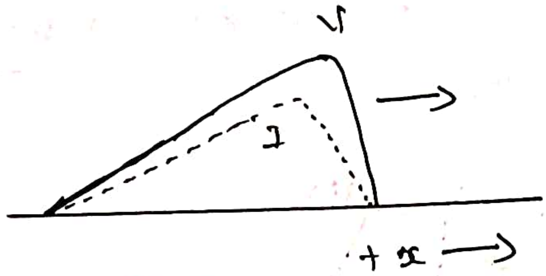


Fig (a): Both V & I (+x direction)

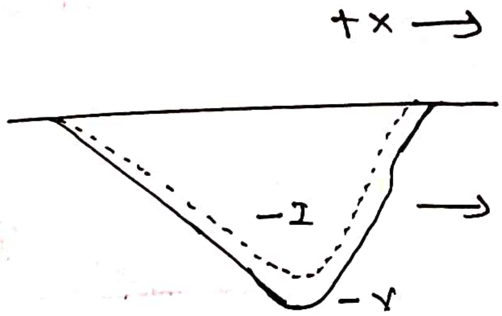


fig (b) (−ve) V & I
(+x direction)

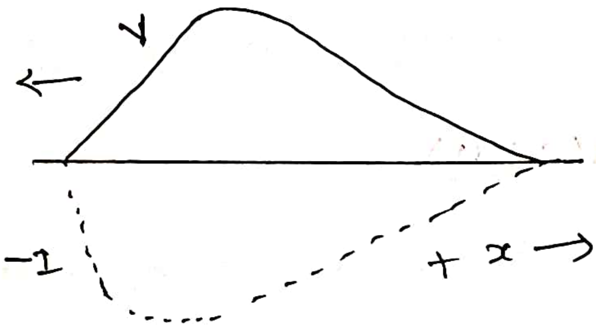


Fig (c)

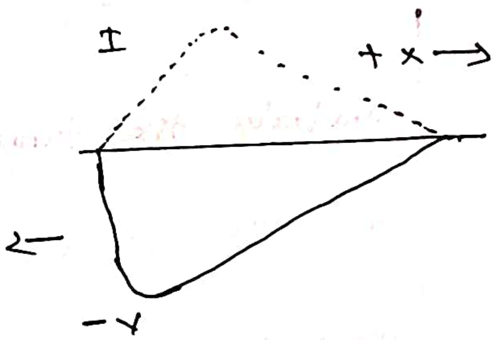


Fig (d)

Fig:- Various combinations of voltage and current waves.

* Above fig shows some combinations of current and voltage waves.
* V & I have the same sign when travelling to the right and opposite signs when travelling to the left

* Fig(c) contains a negative current wave travelling in the negative direction, while fig(d) shows a positive current wave travelling in this direction.

* When two waves travelling in opposite directions meet, they add algebraically as they pass through each other. This is illustrated in fig.

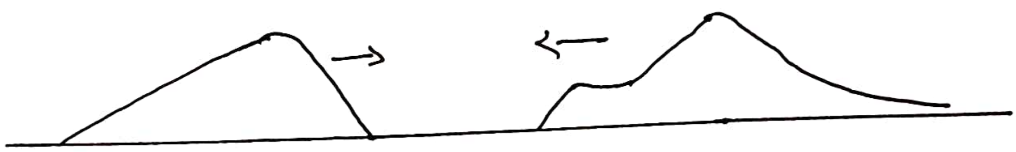
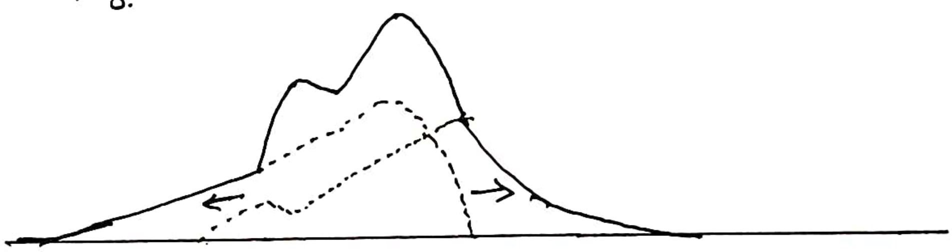


fig:- (a)



fig(b)

fig(c)

fig:- Opposing waves (a) approaching (b) combining (c) passing.

* The waves described by equations (8), (12), (13) & 14 are modified by the reflection and refraction co-efficients appropriate to the next station (or) discontinuity.

* The values of R, Z_1 & β will differ numerically because of the different stations and line characteristics.

* Evidently the product of second ~~term~~ generations encounters. will lead to the transform of the following kind:

$$\frac{1}{(S+\alpha)(S+\beta)(S+\gamma)}, \quad \frac{1}{S(S+\alpha)(S+\beta)(S+\gamma)}, \ldots\ldots$$

———— α ———— α ————x ——x.